An anti-backlash designed planetary gear mechanism: description of general planar motion using matrix methods

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Grant: FR-TI1/594
Název grantu: Výzkum sofistikovaných metod návrhu a vývoje jednoúčelových strojů, komponent a periferií výrobních strojů
Oborové zaměření: JT – Pohon, motory a paliva

Abstract Design of the integrated gear mechanism so arranged that to the three-phase asynchronous motor chassis with a squirrel cage is integrated gear mechanism with internal teeth. In order to eliminate all the backlashes in the gearing and bearings, is branched into two identical gears, between which is inserted the flexible component. The kinematics of bodies, which held general planar motion, are solved by matrix methods of investigation of constrained multibody systems.

Keywords Anti-backlash designed mechanism, planetary gear, vernier, preload

1. INTRODUCTION

In engineering practice, there is often a need for precise positioning of physical objects, such as workpieces, tools, prefabricated components, transported materials, finished products, etc. Positioning device may be linear axis of a machine, rotary positioning table, robotic manipulator and many other cases. In such cases it is necessary to accelerate and brake objects of significant weight. This implies that the drive positioning systems work with relatively large forces and torques, and relatively low speeds. Because the standard designed electric motors usually operate at higher speeds than positioning devices require, it is necessary to insert an appropriate reduction mechanism of rotary motion between motor's rotor and output shaft.

The essence of the concept described in this paper is built backlash-free planetary gear directly to the chassis of the electric motor, with which it forms one construction unit. The gear mechanism, as well as any kinematic mechanism, is made with clearances and tolerances of dimensions. Between the input and output shaft are several such clearances arranged in series, therefore their heights adds. The clearances have a negative effect on the kinematic precision of motion transmission from the input shaft to the output shaft. If the drive is used as a servomotor with a feedback, the backlash implies difficulties also in the regulation of movement. There are usually high requirements to positioning devices for kinematic precision of motion control, therefore the elimination of the backlash is an important technical problem.

2. FUNCTION PRINCIPLE OF THE INTEGRATED GEAR MECHANISM

Design layout of the gearbox is resolved so that a gear with internal toothing is integrated directly to a chassis of electric motor with squirrel cage (fig. 1). In order to eliminate all the backlashes in the gearing and bearings, is branched in-to two identical gears (path A, path B), between which is inserted the flexible component [1], [2]. The paths of transmission are located symmetrically around the the plane of symmetry of the motor. The rotor is equipped at the ends of eccentric cams on which are stored by rolling bearings toothed wheels (satellites). These are in mesh with the central wheels with internal teeth, which are rigid supported in the chassis of the motor. From the satellites is converted rotary motion to output shaft through an annular Oldham couplings and carriers.

Fig. 1. Internal structure of integrated gearbox: 1) stator with stator winding and ring gears, 2) squirrel cage with carriers, 3A) 3B) satellites, 4A), 4B) Oldham couplings, 5) output shaft.
3. BACKLASH ELIMINATION

In order to eliminate all the backlash in the gear mechanism must be assembled in a preloaded condition so that gears have been forced into mesh in opposite directions without external load. During operation, the power is transmitted to either one or the other branches of the transmission, depending on the direction of rotation of the input shaft (rotor). Each branch contains a separate arm of the output shaft. The preload is achieved by torsionally flexible connection between the both arms. The described prototype is designed such that the arms are mounted on a common continuous output shaft. So it is loaded with an additional torque $T_{M}$, which is an optional parameter. However, since the output shaft is designed as a very stiff (in the described prototype it is a full steel shaft with diameter 12 mm and length 151 mm), it is possible to achieve the function of backlash elimination will to choose the value of the preload arbitrarily small. Because the assembly of the whole mechanism is statically indeterminate, the parameter $T_{M}$ must be considered in each computation of force conditions.

The design of setting the torsion preload is solved by turning the ring gears to each other and fixing their positions in the motor chassis. In order to avoid any position change in the operation, it is necessary to use a form-fit joints. In this case, the selected connection pin. However, because the necessary angular position of gear rings continuously adjust, the set of pin holes is drilled around the all perimeter of ring wheels and motor chassis, and it works on the vernier principle. Moreover, the number of holes is bound by the number of teeth ring gears, so that the number of possible positions of gears is very high (see below). In engineering precision scales due to the fact the position is adjustable infinitely smooth (fig. 2).

Furthermore, the table shows the kinematic constraints between individual members and the types of movement that they perform. Since all members perform plane motion in mutually parallel planes, the mechanism is solved as planar.

4. KINEMATIC SCHEME AND GEAR RATIO

Figure 3 shows a kinematic diagram of the gear mechanism. Each transfer branch is composed of members which presents table 1.
5.2 Coordinate transformation

To be able to solve variables in the global coordinate system, which necessary to determine the forces acting on individual members, it is necessary to construct the transformation matrix for the transition between different coordinate systems. Figures 5 to 12 show the relative position of coordinate systems and the corresponding transformation matrices.

The matrix index determines the direction of the transformation. (Example: transformation matrix $T_{4A3A}$ determines transformation $4A \rightarrow 3A$, transformation matrix $T_{4A5}$ determines transformation $3A \rightarrow 4A$.) Matrices are extended, matrices dimension is 4x4. Matrix components at positions $[1,4]$, $[2,4]$ a $[3,4]$ correspond to moving the origin of the coordinate system in the directions $x$, $y$ a $z$. Transformed radiusvectors must therefore be a 4-dimensional.
too. Their term is \( r = [x, y, z]^T \). Because the investigated mechanism is a planar, the coordinate \( z \) is zero. Transformation 2 → 1 has general relation

\[ r^1 = T_{12} r^2. \]  

\[ (4) \]

### 5.3 The kinematic dependences

As the independent variable of all movements was chosen the rotation angle of the rotor with carriers to the stator \( \varphi_2 \). This angle can be any function of time \( \varphi_2 = \varphi_2(t) \). From the kinematic constraints result dependences

\[ \varphi_2 = \varphi_{3,i} = \varphi_{4,i} = \varphi_{4,t} = \frac{-1}{i} \varphi_2, \]  

\[ (5) \]

where \( i \) is gear ratio (equation (3)). The equation (5) fully determines the motion of the output shaft. It remains to solve the general planar motion of the satellites and Oldham couplings.

**Satellite A:** Using basic decomposition of motion it is possible to decompose the satellite A motion to a circular motion and a relative rotary motion. Reference point is \( O_{T1} \). Trajectory of the point is circular with radius \( e \). The circular motion is given by

\[ O_{A,ie} = T_{12} T_{4,12} O_{A,ie}^1. \]  

\[ (7) \]

The equation (7) in a matrix form relates to equation (8).

\[ \begin{bmatrix} \cos \varphi_2 & -\sin \varphi_2 & 0 & 0 \\ \sin \varphi_2 & \cos \varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} \cos \varphi_2 & -\sin \varphi_2 & 0 & 0 \\ \sin \varphi_2 & \cos \varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_2 \\ \varphi_2 \end{bmatrix}. \]  

\[ (8) \]

Substitute equations (5) and (6) and expand (7) to components gives equations for coordinates of point \( O_{A,ie} \) in coordinate system 1:

\[ O_{A,ie} = -e \sin \varphi_2, \]  

\[ (9) \]

\[ O_{A,io} = e \cos \varphi_2. \]  

\[ (10) \]

Relative rotary motion is given by (5). The motion description of satellite B is analogous.

**Oldham coupling A:** Reference point of basic decomposition is the point \( O_{4,ie}^1 = [0,0,0,1]^T \). Moving motion is given by

\[ O_{4,ie} = T_{12} T_{4,12} T_{4,14,1} O_{4,ie}^1. \]  

\[ (11) \]

By multiplication of equations (11) and substitution of (5) and (6) gives the equations for moving motion

\[ O_{A,ie} = X_{O4,14,1} \cos \frac{\varphi_2}{i} - e \sin \varphi_2, \]  

\[ (12) \]

\[ O_{A,io} = -X_{O4,14,1} \sin \frac{\varphi_2}{i} + e \cos \varphi_2. \]  

\[ (13) \]

With the kinematic constraints must also apply the equation

\[ O_{A,ie} = T_{12} T_{4,12} O_{A,ie}^1. \]  

\[ (14) \]

Component equations for position of point \( O_{A,ie} \) in coordinate system 1 are

\[ O_{A,ie} = y_{O4,ie} \sin \frac{\varphi_2}{i}, \]  

\[ (15) \]

\[ O_{A,io} = y_{O4,ie} \cos \frac{\varphi_2}{i}. \]  

\[ (16) \]

From a comparison of the right-hand sides of equations (12), (13), (15) and (16) result a system of two linear equations with a parameter \( \varphi_2 \) for unknown displacement of coordinate systems \( x_{O4,4,14} = x_{O4,4,14} \), \( y_{O4,4,14} = y_{O4,4,14} \), \( x_{O4,4,14} = x_{O4,4,14} \) and \( y_{O4,4,14} = y_{O4,4,14} \). In matrix notation the system has the form

\[ \begin{bmatrix} \cos \frac{\varphi_2}{i} & -\sin \frac{\varphi_2}{i} \\ -\sin \frac{\varphi_2}{i} & -\cos \frac{\varphi_2}{i} \end{bmatrix} \begin{bmatrix} x_{O4,4,14} \\ y_{O4,4,14} \end{bmatrix} = \begin{bmatrix} \sin \varphi_2 \\ -\cos \varphi_2 \end{bmatrix}. \]  

\[ (17) \]

Because the system matrix is symmetric and orthogonal, apply to an unknown displacement relation

\[ \begin{bmatrix} x_{O4,4,14} \\ y_{O4,4,14} \end{bmatrix} = e \begin{bmatrix} \cos \frac{\varphi_2}{i} & -\sin \frac{\varphi_2}{i} \\ -\sin \frac{\varphi_2}{i} & -\cos \frac{\varphi_2}{i} \end{bmatrix} \begin{bmatrix} \varphi_2 \\ \varphi_2 \end{bmatrix}. \]  

\[ (18) \]

Expand (18) to components and modification gives

\[ x_{O4,4,14} = e \sin \left( \frac{\varphi_2 + i}{i} \right), \]  

\[ (19) \]

\[ y_{O4,4,14} = e \cos \left( \frac{\varphi_2 + i}{i} \right). \]  

\[ (20) \]

Substitute (20) to (15) and (16) we get the final expression for the position of the reference point \( O_{A,ie} \) at global coordinate system 1 in the form

\[ \begin{align*}
O_{A,ie} &= e \cos \left( \frac{\varphi_2 + i}{i} \right) \sin \frac{\varphi_2}{i}, \\
O_{A,io} &= e \cos \left( \frac{\varphi_2 + i}{i} \right) \cos \frac{\varphi_2}{i}.
\end{align*} \]  

\[ (21) \]

\[ (22) \]

Relative rotary motion is given by (5). The motion description of Oldham coupling B is analogous.

### 6. CONCLUSION

The aim of paper was to introduce and explore in detail the kinematics of special gear mechanism, which is integrated with the driving electric motor in one unit and thus form a rotary electric actuator. Detailed determination of kinematic variables dependency mechanism is needed for a specific engineering design the drive of this principle.

Further work will mainly cover the design of gear with respect to secondary interference on the heads of teeth and optimize the design to the strength.

Acknowledgements: The research work was made possible by VUTS, a.s. Liberec on research project FR-TI1/594.

References