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New Interactive Multiple Objective Programming Method Applied in the Investment Decision Making under Uncertainty

Adam Borovička[1]

[1] University of Economics, Faculty of Informatics and Statistics, Department of Econometrics, W. Churchill Sq. 4, 130 67 Prague 3; adam.borovicka@vse.cz

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Abstract The article deals with a new interactive multiple objective programming method which is proposed in order to make desirable investment portfolio of open shares funds. Therefore the method takes into account some elements of uncertainty which can occur in the decision making procedures in the capital market. The criteria value may have stochastic character which is solved by an analysis of the generated scenarios. The vague decision maker's preferences about the value of criteria, or their importance, are expressed via the triangular fuzzy numbers. To understand the fuzzy elements in the introduced algorithm, a brief introduction to fuzzy set theory is included. The proposed method is applied in the real case of making the investment portfolio of open shares funds.

Keywords fuzzy set, open shares fund, triangular fuzzy number, uncertainty

1. INTRODUCTION

Many uncertainties can occur in a decision making process, even stochastic character of the criteria values, vague preferences of a decision maker etc. To make the decision making procedure more real, these factors should be included in the used methodical approaches.

So the interactive multiple objective programming method is proposed. This approach uses the fuzzy set theory in order to express the vague, uncertain decision maker's preferences. Thus, the triangular fuzzy number, or the fuzzy number (fuzzy set) with a triangular membership function is employed. The proposed algorithm is also able to take into account a stochastic character of the criteria values by means of a scenario generation. The decision maker plays the active role in the decision making process. He/she evaluates the current solution and suggests its changes if he/she is not satisfied with it. He must mark the value of criterion which would be improved and the values of some criteria that could be worsen any concrete value with some tolerance.

Why is actually new approach proposed? We wanted to solve the particular real decision making situation, namely making the investment portfolio of open shares funds offered by Česká spořitelna Investment Company. We wanted to include some stochastic elements (random variables) and also vague investor's preferences about the values of watched portfolio characteristics. We also expected that the role of investor in the portfolio making is active.

To understand all fuzzy elements in the proposed methods brief introduction to fuzzy set theory is included. At the end of the article, the real investment decision making situation is introduced and solved by the proposed interactive multiple objective programming method.

2. INTRODUCTION TO FUZZY SET THEORY

We always are not able to express anything exactly. A lot of manners are only vague, uncertain. In order to model these situation more precisely, the modified set theory was developed. This concept is known as the fuzzy set theory labouring by L. A. Zadeh (see more Zadeh, 1956).

Let us briefly explain the basic principle of the concept mentioned above. Similar to (Dubois et al., 1980), we denote classical set of the object X called *universe*, whose generic elements are marked x. The membership in a classical subset A of X can be viewed as a membership (or characteristic) function μ_A from X to $\{0,1\}$ such that

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}. \tag{1}$$

$\{0,1\}$ is called *valuation set* which can be also stated as the real interval $\langle 0,1 \rangle$. Then A is called *fuzzy set* (Zadeh, 1965). $\mu_A(x)$ characterizes the grade of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. The fuzzy set A may be written by the set of pairs as follows

$$A = \{(x, \mu_A(x)), x \in X\}. \tag{2}$$

We can say A is a subset of X that has no sharp boundary.

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And now let us introduce two basic operations with fuzzy sets (*intersection* and *union*) by (Pedrycz et al., 2010). Given two fuzzy sets *A*, *B* and their membership functions μ_A, μ_B. The membership function of their intersection $A \cap B$ is computed in the form

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x)) \qquad x \in X \; . \qquad (3)$$

And the membership function of the union $A \cup B$ is determined as follows

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)) \qquad x \in X \; . \qquad (4)$$

2.1 (Triangular) fuzzy number

According to (Dubois et al., 1980), a *fuzzy number* is a convex fuzzy set of the real line *R* such that

a) $\exists!\, x_0 \in R,\ \mu_A(x_0) = 1$ (x_0 is called the *mean value* of *A*),

b) $\mu_A(x)$ is piecewise continuous.

The fuzzy number intuitively represents a value which is inaccurate. This value can be characterized as "about x_0 ". It belongs to very frequent phenomenon in practice.

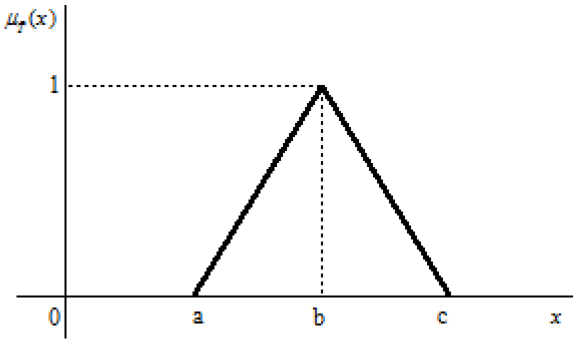
As (Novák, 2000) alludes, the most used type of the fuzzy number is *triangular fuzzy number*. Similar to (Bojadziev et. al, 2007 or Pedrycz, 2010), it can be formalized as

$$\tilde{T} = (a,b,c)\,, \qquad (5)$$

where *b* is the modal value, *a* and *c* is lower and upper bound.

Its membership function has the shape of a triangular as we can see in the following graph (Figure 1).

Figure 1: Membership function of the triangular fuzzy number



Source: (Novák, 2000), self-designed in MS Excel

The membership function of the triangular fuzzy number $\tilde{T}$ is formalized as

$$\mu_{\tilde{T}}(x) = \begin{cases} 0 & x < a \wedge x > c \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 1 & x = b \end{cases} , \qquad (6)$$

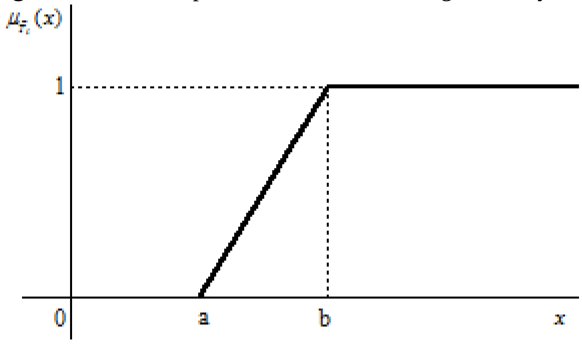
where *a*, *b*, *c* are parameters described and illustrated in figure above. Mostly, the position of parameters *a*, *c* is symmetric around the value of *b*. It means that the membership function usually creates an isosceles triangle.

Sometimes it is needed „only" left or right triangular fuzzy number. As in (Gupta, 2010), the membership function of the left $\tilde{T}_l = (a,b,b)$, or the right $\tilde{T}_r = (b,b,c)$ triangular fuzzy number may be written as follows

$$\mu_{\tilde{T}_l}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x \ge b \end{cases}, \qquad \mu_{\tilde{T}_r}(x) = \begin{cases} 0 & x > c \\ \frac{c-x}{c-b} & b \le x \le c \\ 1 & x \le b \end{cases}. \qquad (7)$$

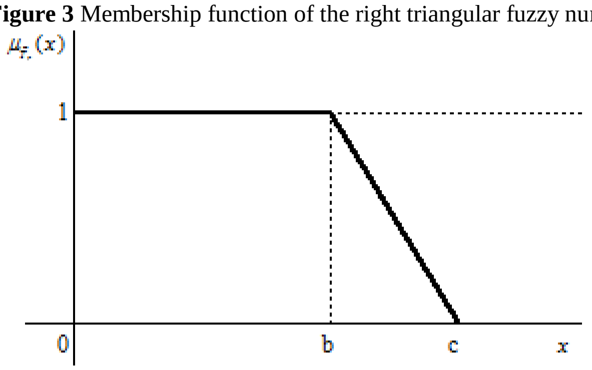
And they can be depicted as in Figure 2 and Figure 3.

Figure 2 Membership function of the left triangular fuzzy number



Source: (Gupta, 2010), self-designed in MS Excel

Figure 3 Membership function of the right triangular fuzzy number



Source: (Gupta, 2010), self-designed in MS Excel

These presented fuzzy sets, or fuzzy numbers will be especially employed in the practical application for investor demands and preferences formulation which sometimes cannot be specified strictly and clearly. We apply the triangular fuzzy numbers in order to express a linear character of investor preferences that can be rationally reflected.

3. NEW INTERACTIVE MULTIPLE OBJECTIVE PROGRAMMING METHOD

The proposed interactive multiple objective programming method takes into account some uncertain elements in the decision making process. Firstly, it accepts a stochastic character of valuation of alternatives by the criteria. Thus, one part of the algorithm is scenario generation and analysis. Secondly, this approach is also able to incorporate decision maker uncertain, vague preferences about the value of criteria (objective functions). These uncertainties are expressed via the triangular fuzzy numbers. Finally, the decision making procedure is interactive, requiring continuous decision maker collaboration.

The algorithm can be described in the several following steps.

Step 1: The model with fuzzy elements
We define all criteria and constraints as a complex problem (Gupta et al., 2010)

.

$$\begin{array}{c} [f_1(x_1,x_2,...,x_n),..., f_k(x_1,x_2,...,x_n)] \to \text{"max"} \\ g_i(x_1,x_2,...,x_n) \geq \tilde{b}_i \quad i=1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq \tilde{b}_i \quad i=m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) = \tilde{b}_i \quad i=m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n) R_i q_i \quad i=1,2,...,r \\ \downarrow \\ [f_1(x_1,x_2,...,x_n),..., f_k(x_1,x_2,...,x_n)] \to \text{"max"} \\ g_i(x_1,...,x_n) \geq b_i - b_i^* \quad i=1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^* \quad i=m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) \geq b_i - b_i^l \quad i=m_2+1,2,...,m \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^u \quad i=m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n) R_i q_i \quad i=1,2,...,r \end{array}, \tag{7}$$

where $f_l\ (l=1,2,...,k)$ expresses the *l-th* objective function (criterion), $x_j\ (j=1,2,...,n)$ represents the *j-th* unknown variable, $g_i\ (i=1,2,...,m)$ is the left side and $\tilde{b}_i\ (i=1,2,...,m)$ is the right side of the *i-th* limit. The values $\tilde{b}_i$ show an uncertainty, so the decision maker (DM) does not determine the strict demands, but only the b_i level with tolerance $b_i^*\ (i=1,2,...,m_2)$, or b_i^l and $b_i^u\ (i=m_2+1,...,m)$ according to the constraint type. The symbol $p_i\ (i=1,2,...,r)$ is the left side, $R_i\ (i=1,2,...,r)$ a relational sign, and $q_i\ (i=1,2,...,r)$ represents the right side of the *i-th* limit with no fuzzy elements. This fact is shown in the model on the right side brightly quantifying the vague requirements in the extreme tolerance concept. It is desirable to reach values as high as possible of all objective functions ("max").

Step 2: The single criterion model of linear programming

Set the lower L_l and upper U_l bound for the *l-th* objective. To calculate these bounds of all objective functions we first solve the following sub problems for each *i-th* objective function of minimizing or maximizing character.

$$\begin{array}{c} z_l = f_l(x_1,x_2,...,x_n) \to \max(\min) \\ g_i(x_1,...,x_n) \geq b_i \quad i=1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i \quad i=m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) = b_i \quad i=m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n) R_i q_i \quad i=1,2,...,r \\ \downarrow \\ z_l = f_l(x_1,x_2,...,x_n) \to \max(\min) \\ g_i(x_1,...,x_n) \geq b_i - b_i^* \quad i=1,2,...,m_1 \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^* \quad i=m_1+1,2,...,m_2 \\ g_i(x_1,x_2,...,x_n) \geq b_i - b_i^l \quad i=m_2+1,2,...,m \\ g_i(x_1,x_2,...,x_n) \leq b_i + b_i^u \quad i=m_2+1,2,...,m \\ p_i(x_1,x_2,...,x_n) R_i q_i \quad i=1,2,...,r \end{array}. \tag{8}$$

We can identify the optimal solution of the first model as $\mathbf{x}_{1j}^o$, or the second one $\mathbf{x}_{2j}^o\ (j=1,2,...,k)$ with the values of objective functions $z_l^o(\mathbf{x}_{1j}^o)$, or $z_l^o(\mathbf{x}_{2j}^o)$ for $j,l=1,2,...,k$. Then the lower (L_l) and upper (U_l) bounds of the *l-th* objective function are calculated as follows

$$L_l = \min\{z_l^o(\mathbf{x}_{1j}^o), z_l^o(\mathbf{x}_{2j}^o)\} \qquad U_l = \max\{z_l^o(\mathbf{x}_{1j}^o), z_l^o(\mathbf{x}_{2j}^o)\}. \tag{9}$$

When the aspiration levels for each objective are obtained, we can form a fuzzy model where find $x_j\ (j=1,2,...,n)$ so as to satisfy

$$\begin{array}{l} z_l \tilde{\leq} L_l \quad \forall l(\min) \\ z_l \tilde{\geq} U_l \quad \forall l(\max) \\ g_i(x_1,...,x_n) \tilde{\geq} b_i \quad i=1,2,...,m_1 \\ g_i(x_1,...,x_n) \tilde{\leq} b_i \quad i=m_1+1,...,m_2 \\ g_i(x_1,...,x_n) \tilde{=} b_i \quad i=m_2+1,...,m \\ p_i(x_1,x_2,...,x_n) R_i q_i \quad i=1,2,...,r \end{array}. \tag{10}$$

The membership functions (see more Klir et al., 1995) for fuzzy constraints of (10) are defined as a form of the triangular fuzzy number. Similar to (Gupta et al., 2010), for the *l-th* constraints (G_l) according to minimizing or maximizing objective function, the following holds

$$\mu_{G_l}(z_l) = \begin{cases} 1 & z_l \leq L_l \\ \frac{U_l - z_l}{U_l - L_l} & L_l \leq z_l \leq U_l \\ 0 & z_l > U_l \end{cases} \quad \mu_{G_l}(z_l) = \begin{cases} 1 & z_l \geq U_l \\ \frac{z_l - L_l}{U_l - L_l} & L_l \leq z_l \leq U_l \\ 0 & z_l < L_l \end{cases}. \tag{11}$$

For the *i-th* constraint (E_i), where $i=1,2,...,m_1$, $i=m_1+1,...,m_2$, or $i=m_2+1,...,m$ in agreement with the type of limit, we can write the membership function in the mentioned order

$$\begin{array}{l} \mu_{E_i}(b_i) = \begin{cases} 1 & g_i(\mathbf{x}) \geq b_i \\ \frac{g_i(\mathbf{x}) - b_i + b_i^o}{b_i^o} & b_i - b_i^o \leq g_i(\mathbf{x}) \leq b_i \\ 0 & g_i(\mathbf{x}) < b_i - b_i^o \end{cases} \\ \mu_{E_i}(b_i) = \begin{cases} 1 & g_i(\mathbf{x}) \leq b_i \\ \frac{b_i + b_i^o - g_i(\mathbf{x})}{b_i^o} & b_i \leq g_i(\mathbf{x}) \leq b_i + b_i^o \\ 0 & g_i(\mathbf{x}) > b_i + b_i^o \end{cases} \\ \mu_{E_i}(b_i) = \begin{cases} 0 & g_i(\mathbf{x}) < b_i - b_i^l \\ \frac{g_i(\mathbf{x}) + b_i^l - b_i}{b_i^l} & b_i - b_i^l \leq g_i(\mathbf{x}) \leq b_i \\ \frac{b_i^u + b_i - g_i(\mathbf{x})}{b_i^r} & b_i \leq g_i(\mathbf{x}) \leq b_i + b_i^u \\ 0 & g_i(\mathbf{x}) > b_i + b_i^u \end{cases} \end{array}. \tag{12}$$

According to (Černý et al., 1987), the fuzzy decision is represented by the fuzzy set $A = G_1 \cap ... \cap G_k \cap E_1 \cap ... \cap E_m \cap X$, where X is a (non-fuzzy) set of feasible solutions of the initial problem, thus $X = \{x \in R^n, p_i(x) R_i q_i, i=1,2,...,r\}$. The optimal solution $\mathbf{x}^* \in X$ has the maximum value of membership function $\mu_A = \min_{i,l}(\mu_{G_l}(z_l), \mu_{E_i}(b_i))$. As in (Černý et al., 1987), the optimal solution can be obtained via the problem of linear programming written as follows

$$\begin{array}{l} \lambda \to \max \\ z_l + \lambda(U_l - L_l) \leq U_l \quad \forall l(\min) \\ z_l - \lambda(U_l - L_l) \geq L_l \quad \forall l(\max) \\ g_i(x_1,...,x_n) - \lambda b_i^o \geq b_i - b_i^o \quad i=1,2,...,m_1 \\ g_i(x_1,...,x_n) + \lambda b_i^o \leq b_i + b_i^o \quad i=m_1+1,...,m_2 \\ g_i(x_1,...,x_n) - \lambda b_i^o \geq b_i - b_i^l \quad i=m_2+1,...,m \\ g_i(x_1,...,x_n) + \lambda b_i^o \leq b_i + b_i^u \quad i=m_2+1,...,m \\ p_i(x_1,...,x_n) R_i q_i \quad i=1,2,...,r \\ 0 \leq \lambda \leq 1 \end{array}, \tag{13}$$

where $\lambda = \min_{i,l}\{\mu_{G_l}(z_l), \mu_{E_i}(b_i)\}$.

In the case of the fuzzy weights of criteria, the optimal solution $x^* \in X$ has the maximum value of the membership function $\mu_A = \min_{i,l}\{\mu_{W_l}(\mu_{G_l}(z_l)), \mu_{E_i}(b_i)\}$, where μ_{W_l} represents the membership functions describing the fuzzy decision maker preferences about the criteria.

Step 3: **Interactive procedure**
When the current solution is acceptable, the process is finished. If not, the decision maker (investor) has some demands for solution (portfolio) improvement that can have a fuzzy character, so some additional constraints will be included in the model. We select the criteria that should be improved, then new constraints are as follows

$$z_l \geq z_l^c + \Delta z_l \quad \forall l(\max) \qquad z_l \leq z_l^c - \Delta z_l \quad \forall l(\min), \qquad (14)$$

where $\Delta z_l\ (l = 1, 2, ...k)$ expresses the desired minimal betterment of the *l-th* criterion and $z_l^c\ (l = 1, 2, ..., k)$ is the current value of the *l-th* objective function. Under these conditions the solution can be infeasible. Then DM has to shrink his or her demands to find it. It is obvious that values of some other criteria must be sacrificed. The DM accepts the decrease value of maximizing, or the increase value of minimizing criterion in the value $\Delta^{\max}$ with a tolerance $\bar{\Delta}^{\max}$, or $\Delta^{\min}$ with a tolerance $\bar{\Delta}^{\min}$, then

$$\forall l(\max) \quad z_l \gtrsim z_l^c - \Delta^{\max} \rightarrow z_l^c - z_l \lesssim \Delta^{\max} \rightarrow z_l^c - z_l \leq \Delta^{\max} + \bar{\Delta}^{\max} \quad (15)$$

(with extreme tolerance),

$$\forall l(\min) \quad z_l \lesssim z_l^c + \Delta^{\min} \rightarrow z_l - z_l^c \lesssim \Delta^{\min} \rightarrow z_l - z_l^c \leq \Delta^{\min} + \bar{\Delta}^{\min} \quad (16)$$

(with extreme tolerance).

Now the membership function for new preference constraints may be declared as

$$\mu_B(\Delta^{\max}) = \begin{cases} 1 & z_l^c - z_l \leq \Delta^{\max} \\ \frac{\Delta^{\max} + \bar{\Delta}^{\max} - (z_l^c - z_l)}{\bar{\Delta}^{\max}} & \Delta^{\max} \leq z_l^c - z_l \leq \Delta^{\max} + \bar{\Delta}^{\max} \\ 0 & z_l^c - z_l > \Delta^{\max} + \bar{\Delta}^{\max} \end{cases},$$
$$\mu_B(\Delta^{\min}) = \begin{cases} 1 & z_l - z_l^c \leq \Delta^{\min} \\ \frac{\Delta^{\min} + \bar{\Delta}^{\min} - (z_l - z_l^c)}{\bar{\Delta}^{\min}} & \Delta^{\min} \leq z_l - z_l^c \leq \Delta^{\min} + \bar{\Delta}^{\min} \\ 0 & z_l - z_l^c > \Delta^{\min} + \bar{\Delta}^{\min} \end{cases}. \qquad (17)$$

So we must add particular constraints representing fuzzy preferences in the final model in the following form

$$z_l^c - z_l + \lambda\bar{\Delta}^{\max} \leq \Delta^{\max} + \bar{\Delta}^{\max} \qquad z_l - z_l^c + \lambda\bar{\Delta}^{\min} \leq \Delta^{\min} + \bar{\Delta}^{\min}. \quad (18)$$

The third step is repeated until the solution is acceptable for the decision maker.

4. INVESTMENT DECISION MAKING UNDER UNCERTAINTY

The potential investor decided to invest some money in the open shares funds offered and managed by Česká spořitelna Investment Company. He chooses from four groups - money-market funds[1], mixed funds, bond funds and stock funds as the following table closely shows (Table 1).

Table 1: List of shares funds offered by Česká spořitelna Investment Company[2]

Money-market funds	Mixed funds	Bond funds	Stock funds
Sporoinvest	Osobní portfolio 4 Plus Fond řízených výnosů Konzervativní Mix Vyvážený Mix Dynamický Mix Akciový Mix	Sporobond Trendbond Bondinvest Korporátní dluhopisový High Yield dluhopisový	Sporotrend Global Stocks Top Stocks

The investor follows two criteria – return and risk. Further costs and Sharpe ratio also play a big role in the decision making process. The return is a random variable with a normal probability distribution. The criterion risk is more important than the return for investor which can be seen in the interactive decision making procedure. The highest possible level of costs is stated as 2 % with tolerance 0.5 %, the lowest level of Sharpe ratio 0.2 % with 0.2 % tolerance to cover at least the risk-free yield rate. Further the investor requires the minimum share of one shares fund 5 % and the maximum level 50 % with the view of portfolio diversification.

Firstly, one fund from each group is selected via some multiple criteria evaluation method (see more Borovička, 2012). The choice shares funds are *Dynamický Mix*, *Sporoinvest*, *Bondinvest*, *Global Stocks*. To make the investment portfolio of chosen shares funds, the proposed interactive multiple objective programming method is applied. Before that, we generate 100 scenarios of returns, so we get 100 aspiration levels of all objective functions. The final mathematical model is formulated as follows

$$\lambda \rightarrow \max$$
$$\begin{array}{ll} \sum_{j=1}^{4} v_j x_j - \lambda(U_1 - L_1) \leq L_1 & x_j \geq 0.05 \quad j = 1,...,4 \\ \sum_{j=1}^{4} r_j x_j + \lambda(U_2 - L_2) \geq U_2 & x_j \leq 0.5 \quad j = 1,...,4 \\ \sum_{j=1}^{4} n_j x_j + 0.5\lambda \leq 2.5 & \sum_{j=1}^{4} x_j = 1 \\ \sum_{j=1}^{4} s_j x_j - 0.2\lambda \geq 0 & 0 \leq \lambda \leq 1 \end{array}, \qquad (19)$$

where $v_j, r_j, n_j, s_j\ (j = 1, ..., 4)$ is return, risk, costs and Sharpe ratio of the j-th shares fund, $x_j\ (j = 1, ..., 4)$ represents a share of the j-th fund in a portfolio. The values L_1 and U_1, or L_2 and U_2 are the aspiration levels of the particular objective function.

We choose the solution with the biggest value of the objective function for an interactive procedure. Its shape is: 22.6 % Dynamický Mix, 22.4 % Sporoinvest, 50 % Bondinvest, 5 % Global Stocks with 1.28 % return and 2.03 % risk, λ = 0.59. The objective function of the model (lambda) represents the membership grade of the solution. It is obvious that it will decrease during the procedure of making portfolio changes. As the portfolio risk is more important than return, so the investor wishes to make better the value of risk. He/She is willing to accept a decrease the portfolio return by 0.1 %

[1] Nowadays, the group of money-market open shares funds does not exist. The shares fund Sporoinvest is implicated in the group of bond open shares funds.

[2] http://www.iscs.cz/web/fondy (cit. 30. 12. 2012). In recent time, the fund Bondinvest was already removed from the offer.

with the same tolerance, but he demands the risk under 1.9 % level. Thus two following constraints must be added in the model

$$\begin{aligned} &\sum_{j=1}^{4} r_j x_j \le 1.9 \\ &1.28 - \sum_{j=1}^{4} v_j x_j + 0.1\lambda \le 0.2 \end{aligned} \quad . \tag{20}$$

The next solution is: 48.1 % Dynamický Mix, 41.9 % Sporoinvest, 5 % Bondinvest, 5 % Global Stocks with 1.2 % return and 1.9 % risk, λ = 0.55. The investor still wants to decrease the risk at the expanse of return, below 1.7 % level with the same acceptable decrease return as in the previous case. After the model is changed by the supplements

$$\begin{aligned} &\sum_{j=1}^{4} r_j x_j \le 1.7 \\ &1.2 - \sum_{j=1}^{4} v_j x_j + 0.1\lambda \le 0.2 \end{aligned} \quad . \tag{21}$$

The solution with the values of portfolio characteristics looks as in the following tables (Table 2, Table 3).

Table 2: Final investment portfolio structure

Shares fund	**Share**
Dynamický Mix	41.1 %
Sporoinvest	48.9 %
Bondinvest	5 %
Global Stocks	5 %

Source: own

Table 3: Values of the objective function and the final portfolio characteristics

λ	0.5
Risk	1.7 %
Return	1.08 %

Source: own

The next demand on the risk cut-down about 0.2 % is not acceptable because of solution infeasibility. The investor agrees with the prior one (see Table 2).

As we can see, the main part of the investment portfolio is created by Sporoinvest and Dynamický Mix. The money-market and mixed fund give lower level of risk which is the most important characteristic for the investor. Then other two shares funds participate in the portfolio by the lowest level of share. These funds represent more risky investment alternatives.

5. Conclusion

The goal of the article was an introduction to new interactive multiple objective programming method. The proposed approach can take into account some stochastic and vague, uncertain elements in the decision making process. The criteria values may be set as random variables, decision maker's preferences can be expressed blankly. The method uses Monte Carlo optimization and also the triangular fuzzy numbers in order to express these possible uncertainties. The role of the decision maker is active. He participates in the procedure of a making final solution.

This method is proposed in order to give a hand with making an appropriate investment decision. Particular application of this approach is described in the final part of the paper, where potential investor wants to make an investment portfolio of the open shares funds.

In the end it is necessary to remind that investment decision making is based on the historical data about open shares funds. It is not possible to ensure that the future development of portfolio characteristics will be the same as in the past. But this is the well-known phenomenon, because any predictions in the field of capital market are so difficult. The investor should take into account this fact and eventually make another analysis, for example about actual situation or mood in the capital market etc.

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