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On the Cross-Correlation Properties of Two-Dimensional Complete Complementary Codes of Different Orders for (N, N, N2×N2)

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Abstract Since invention of Complete Complementary Codes (CCC) those were attracting researchers' attention due to their ideal auto- and cross-correlation properties. Recently, new ways of their automated constructions were presented. Due to limitations given by constructions in concurrent number of signatures various approaches were designed in order to allow higher number of concurrent transmissions. Most recent approach aiming increase of signature number was introduced by Suehiro in which elements of various lengths of one-dimensional CCCs are analysed. By using two-dimensional (2D) CCCs diversity is achieved. Existing constructions of 2D-CCCs deal solely with elements of same order. In this paper discussion about 2D-CCCs of various element dimensions is described. This enables higher channel efficiency and thus higher transmission rates and capacity.

Keywords Complete complementary code, CCC, mutually orthogonal complementary set, two-dimensional

1. INTRODUCTION

Invention of Complete complementary codes (CCC) enabled their application in many different areas of digital technology, but particularly in telecommunication systems. In [6] Golay firstly published complementary codes in relation with optical infrared, multi-slit spectrometry. Further development was achieved through complementary sequences later in [5], [11], [13], [15]. Constructions published in mentioned papers were mostly concerned only with autocorrelation properties of resulting sequences. First sets of sequences with both ideal correlation properties, thus ideal autocorrelation and ideal cross-correlation, were proposed by Tseng and Liu [14]. These sets were called Complete Complementary Codes (CCC) afterwards [5], [11] and [13]. Development of CCC generating algorithm still represents a vivid area of research. The most recent publication in this field proposes a framework for systematic construction of these codes [7].

Subsequently authors in [2] suggested construction of two-dimensional complete complementary codes (2D-CCC) with minimum element order of N2. Following research resulted in generalizing the construction given in [2] which enabled acceptance of any of the recent one-dimensional CCCs [7], [12] and [17] on input. This leads to construction of 2D-CCC with elements of smaller order. As a result higher number of concurrently transmitted signatures was possible and consequently it followed to increase the channel capacity [1].

In this paper another approach aiming to increase channel capacity is discussed. This approach consists of using signatures of various

orders concurrently. All previous contributions concerning 2D-CCC were using solely signatures of same order. The new approach connects on proposal in [7] where systematic construction of 1D-CCC of different lengths was proposed. In 2D the impact of different orders is different from 1D case.

The paper is organized as follows: An introduction to CCC is given in section II. In section III the generalized construction of analyzed codes is given. In section IV the cross-correlation properties of 2D-CCCs of different orders are discussed. In Conclusion V the advantages of proposed approach are summarized.

2. CCC DEFINITIONS

2.1 1D-CCC

Discrete aperiodic cross-correlation function $C_{p,r}$ [10] is given as follows:

$$C_{p,r}(s)=\begin{cases}\sum_{u=0}^{T-1-s} a_u^{(p)} a_{u+s}^{(r)}, & 0\le s\le T-1\\ \sum_{u=0}^{T-1+s} a_u^{(p)} a_{u+s}^{(r)}, & 1-T\le s<0\\ 0, & |l|\ge T\end{cases} \tag{1}$$

where s denotes the shift, T denotes the period of equally long distinguish sequences $(a_j^{(p)})$ and $(a_j^{(r)})$ of p-th and r-th users consisting of coordinates +1 and -1. If $p=r$, $C_{p,r}$ denotes discrete aperiodic autocorrelation function.

Signature denotes a collection of sequences assigned to one user where autocorrelation function computed through all sequences is zero for any nonzero shift. *Mutually orthogonal signatures* are two signature sets where every two complementary sets in the collections are mates of each other.

CCC signature sequences have to be transmitted via separate channels [8]. This allows computing the auto- and cross-correlation at receiver independently for each channel and further it follows summarization of the results in order to obtain overall cross- and autocorrelation. This enables achievement of ideal cross- and autocorrelation properties.

Due to above reasons original definition of aperiodic autocorrelation [10] has to be slightly modified in such a way that computation is done through all sequences for a given signature:

The *aperiodic autocorrelation* function $\rho_{\mathbf{c}^{(i)}}$ of L long sequence $c^{(i)}$ is defined as follows:

$$\rho_{\mathbf{c}^{(i)}}(\tau)=\sum_{l=0}^{L-1}\mathbf{c}^{(i)}(l).\left[\mathbf{c}^{(i)}(l+\tau)\right]^* \tag{2}$$

where τ denotes the shift.

The *aperiodic cross-correlation function* $\rho_{\mathbf{c}^{(i)},\mathbf{c}^{(j)}}(\tau)$ between two different sequences $c^{(i)}\in C$ and $c^{(j)}\in C$ where $i\neq j$:

$$\rho_{\mathbf{c}^{(i)},\mathbf{c}^{(j)}}(\tau)=\sum_{l=0}^{L-1}\mathbf{c}^{(i)}(l).\left[\mathbf{c}^{(j)}(l+\tau)\right]^* \tag{3}$$

i-th signature in a set C of N signatures is denoted as follows:

$$\mathbf{c}^{(i)}=(\mathbf{c}_1^{(i)}\quad \mathbf{c}_2^{(i)}\quad \dots\quad \mathbf{c}_E^{(i)});\quad i=1,2,\dots N \tag{4}$$

where each sequence:

$$\mathbf{c}_k^{(i)}=(c_{k,1}^{(i)}\quad c_{k,2}^{(i)}\quad \dots\quad c_{k,L}^{(i)});\quad k=1,2,\dots E \tag{5}$$

is a k-th element of a signature with length L. Element is a vector composed of coordinates, thus symbols (usually complex numbers with amplitude one). For convenience of the reader coordinates ± 1 are used in this article.

CCC have ideal aperiodic auto- and cross-correlation properties, thus (2) and (3) are equal to zero except for zero shift of aperiodic autocorrelation (2).

The most recent contribution [9] proposed 1D-CCC construction of element length L which is equal to maximal number of signatures N. Other recent published methods lead to alternative values of L, namely L equal to e.g. N^2, 2^mN or N using construction methods in [9], [12] and [17], respectively, where N is power of two. In practical applications minimizing L [8] whilst maximizing N is sometimes desired.

2.2 2D-CCC

Let $\mathbf{C}$ be a complex matrix of order P made of complex numbers c_{ij} whose absolute values $|c_{ij}|=1$:

$$\mathbf{C}=\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1P}\\ \dots & & & \\ c_{P1} & c_{P2} & \dots & c_{PP}\end{bmatrix} \tag{6}$$

M_{2D} sets of N_{2D} matrices

$$\left\{\mathbf{C}_1^{(1)},\mathbf{C}_2^{(1)},\dots\mathbf{C}_{N_{2D}}^{(1)}\right\},\ \dots\ \left\{\mathbf{C}_1^{(M_{2D})},\mathbf{C}_2^{(M_{2D})},\dots\mathbf{C}_{N_{2D}}^{(M_{2D})}\right\} \tag{7}$$

compose a 2D-CCC of order M_{2D} with autocorrelation function is zero except for zero shift (computed through all elements of given signature) and cross-correlation is zero between two different signatures (computed through all elements). The auto- and cross-correlation functions are defined in Appendix A (13) and (14), respectively. A set of N_{2D} matrices { $\mathbf{C}^{(i)}{}_1$, $\mathbf{C}^{(i)}{}_2$,…, $\mathbf{C}^{(i)}{}_{N2D}$ } is termed i-th signature of 2D-CCC. A matrix $\mathbf{C}^{(i)}{}_j$ is termed the j-th element of i-th signature.

3. CONSTRUCTION OF 2D-CCC [1]

In [1] the following construction was proposed. Let $\mathbf{C}_{n_{2D}}^{(k_{2D})}$ denote the n_{2D}-th element of the k_{2D}-th signature of the new 2D-CCC. It is a matrix with order P. Let $\mathbf{c}_{n_{2D},i}^{(k_{2D})}$ denote its i-th row $i=1,2,\dots,P$, $P=L$.

Each element of 2D-CCC signature is composed of rows obtained using following equation:

$$\mathbf{c}_{n_{2D},i}^{(k_{2D})}=\mathbf{c}_{\left[(n_{2D}-1)\bmod N_{1D}\right]+1}^{\left[(k_{2D}-1)\bmod M_{1D}\right]+1}\times c_{v,i}^{(t)} \tag{8}$$

$$k_{2D}=1,2,\dots,M_{1D}{}^2 \tag{9}$$

$$n_{2D}=1,2,\dots,N_{1D}{}^2 \tag{10}$$

$$v=\left\lfloor\frac{n_{2D}-1}{N_{1D}}+1\right\rfloor,\ t=\left\lfloor\frac{k_{2D}-1}{M_{1D}}+1\right\rfloor \tag{11}$$

where $\lfloor x \rfloor$ is the greatest integer, which is equal or smaller than x, M_{1D} and N_{1D} denote number of signatures and elements of inputted 1D-CCC, respectively.

4. CROSS-CORRELATION PROPERTY OF SIGNATURES WITH DIFFERENT ORDERS

In 1D-CCC it is possible to use signatures with elements of different lengths [9] where the cross-correlation property remains ideal for any shift if those signatures are from different families. In 2D-CCC this also can be achieved for signatures of different order. This fact has been gained by computing of various selected combinations of signatures. However, the order of given signatures varies.

For 2D-CCC based on a Walsh-Hadamard matrix of order 4 used for input of 1D-CCC creation according to algorithm published in [9], we get the following triple of basic parameters *(4,4,4)*. Next using the corresponding results for creation of 2D-CCC according to algorithm published in [1], we get a *(4,4,16×16)* code. The test results shown in Tab.1 were achieved when computing cross-correlation properties with *(8,8,64×64)* code generated with the same algorithm. The first column shows signatures of the first code with greater order and the second column lists signatures of the second code with smaller order where the cross-correlation function with the signature of the first code resulted in ideal values, thus in zero values for all possible shifts.

TABLE I 1st code *(8,8,64×64)*, 2nd code *(4,4,16×16)*

Sig. 1	Signatures of 2nd code with ideal XCF											
1	2	3	4	6	7	8	10	11	12	14	15	16
2	1	3	4	5	7	8	9	11	12	13	15	16
3	1	2	4	5	6	8	9	10	12	13	14	16
4	1	2	3	5	6	7	9	10	11	13	14	15
5	2	3	4	6	7	8	10	11	12	14	15	16
6	1	3	4	5	7	8	9	11	12	13	15	16
7	1	2	4	5	6	8	9	10	12	13	14	16
8	1	2	3	5	6	7	9	10	11	13	14	15
…	…											

From the results it is obvious that the 1st signature of 1st code cannot be used with the 1st, 5th, 9th, 13th signature of the 2nd code. Thus, each 4th signature has to be left out when starting with 1st. This is also valid for the 2nd signature of 1st code starting with 2nd signature, thus not possible to use with 2nd, 6th, 10th, 14th.

This pattern is repeating for each 4th signature of 1st code as it can be seen in the Tab.1, thus 1st, 5th, 9th, 13th, … cannot be used with the same signatures of 2nd code as it is valid for the 1st signature of 1st code.

Cross-correlation properties of code combination in which the 1st code is *(4,4,16×16)* and 2nd code is *(2,2,4×4)* are shown in the Tab.2. We can see that each signature of 1st code can be used with solely two signatures of 2nd code in the following manner: each second signature of 1st code can be used solely with 2nd and 4th signature of 2nd code when starting with 1st signature. All remaining signatures of 1st code are to be used with 1st and 2nd signature of 2nd code in order to achieve ideal cross-correlation properties.

TABLE 2 1st code *(4,4,16×16)*, 2nd code *(2,2,4×4)*

Signature of 1st code	Signatures of 2nd code with ideal XCF
1	2 4
2	1 3
3	2 4
4	1 3
5	2 4
6	1 3
7	2 4
8	1 3
9	2 4
10	2 4
11	1 3
12	2 4
13	1 3
14	2 4
15	1 3
16	2 4

This pattern can be seen also for combinations of greater orders of 2D-CCC codes.

By further analysis (calculating cross-correlation function of codes with different order based on above mentioned generating constructions) it can be seen that the number of signatures with smaller order which could be used is given by the total amount of signatures of smaller order divided by the order itself.

5. ADVANTAGES

Using 2D-CCC in two dimensional time-frequency domain as proposed in [3]-[4] resulting in Multicarrier Code Division Multiple Access (MC-CDMA). Usage of signatures with different order of 2D elements allows their better adaptation to available frequency domain during transmission and so the 2D channels could be better exploited, what results in higher throughput.

In MC-CDMA application the number of concurrently communicating users is given by number of signatures, because each user has to be assigned at least one signature, where each signature consists of equal number of elements. In case of an optimal CCC, this number of elements composing one signature is equal to total the number of signatures.

The number of elements gives the number of separate channels needed for transmission because each element of one signature has to be transmitted via separate channel [8].

In case of transmission of signatures with the same element orders, the corresponding elements of different signatures are transmitted via same channel.

In case of transmission of signatures with different orders, the optimality of the cross-correlation property has to be maintained. As it was shown in this article, this can be achieved by using combinations with ideal cross-correlation properties. Transmission of codes with different element order enables usage of more users concurrently. E.g. one user is assigned higher element order enables serving more users concurrently. E.g. one user is assigned higher element order and at least two users are transmitting via the same channel with smaller order concurrently.

6. CONCLUSION

In this paper results of cross-correlation properties analysis for 2D-CCC were presented in table.

As shoved the cross-correlation property remains ideal also for signatures of different order, however, for different order combinations of given signatures. Usage of signatures with different order enables higher number of concurrently transmitting signatures, thus increase in channel throughput as discussed in chapter 3 of this paper.

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Appendix A

Two-Dimensional Correlation Functions

$$\boldsymbol{\rho}(\mathbf{C},o,p)=\begin{cases}\frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1}^{N-p}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\ o=0,1,\dots M-1;\ p=0,1,\dots N-1\\ \frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1-p}^{N}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\ o=0,1,\dots M-1;\ p=-N+1,\dots -1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1}^{N-p}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\ o=-M+1,\dots -1;\ p=0,1,\dots N-1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1-p}^{N}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\ o=-M+1,\dots -1;\ p=-N+1,\dots -1\end{cases};\quad (13)$$

where $\mathbf{c}^{*}_{ij}$ is the complex conjugate of $\mathbf{c}_{ij}$.

$$\boldsymbol{\rho}(\mathbf{C}^{(x)},\mathbf{C}^{(y)},o,p)=\begin{cases}\frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1}^{N-p}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)\,*}_{(k+o)(l+p)} & for\ o=0,1,\dots M-1;\ p=0,1,\dots N-1\\ \frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1-p}^{N}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)\,*}_{(k+o)(l+p)} & for\ o=0,1,\dots M-1;\ p=-N+1,\dots -1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1}^{N-p}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)\,*}_{(k+o)(l+p)} & for\ o=-M+1,\dots -1;\ p=0,1,\dots N-1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1-p}^{N}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)\,*}_{(k+o)(l+p)} & for\ o=-M+1,\dots -1;\ p=-N+1,\dots -1\end{cases};\quad (14)$$

where $\mathbf{c}^{*}_{ij}$ is the complex conjugate of $\mathbf{c}_{ij}$.