# An anti-backlash designed planetary gear mechanism: description of general planar motion using matrix methods 

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#### Abstract

Design of the integrated gear mechanism so arranged that to the three-phase asynchronous motor chassis with a squirrel cage is integrated gear mechanism with internal teeth. In order to eliminate all the backlashes in the gearing and bearings, is branched into two identical gears, between which is inserted the flexible component.The kinematics of bodies, which held general planar motion, are solved by matrix methods of investigation of constrained multibody systems.


Keywords Anti-backlash designed mechanism, planetary gear, vernier, preload

## 1. INTRODUCTION

In engineering practice, there is often a need for precise positioning of physical objects, such as workpieces, tools, prefabricated components, transported materials, finished products, etc. Positioning device may be linear axis of a machine, rotary positioning table, robotic manipulator and many other cases. In such cases it is necessary to accelerate and brake objects of significant weight. This implies that the drive positioning systems work with relatively large forces and torques, and relatively low speeds. Because the standard designed electric motors usually operate at higher speeds than positioning devices require, it is necessary to insert an appropriate reduction mechanism of rotary motion between motor's rotor and output shaft.

The essence of the concept described in this paper is built backlashfree planetary gear directly to the chassis of the electric motor, with which it forms one construction unit. The gear mechanism, as well as any kinematic mechanism, is made with clearances and tolerances of dimensions. Between the input and output shaft are several such clearances arranged in series, therefore their heights adds. The clearances have a negative effect on the kinematic precision of motion transmission from the input shaft to the output shaft. If the drive is used as a servomotor with a feedback, the backlash implies difficulties also in the regulation of movement. There are usually high requirements to positioning devices for kinematic precision of motion control, teherefore the ellimination of the backlash is an important technical problem.

## 2. FUNCTION PRINCIPLE OF THE INTEGRATED GEAR MECHANISM

Design layout of the gearbox is resolved so that a gear with internal toothing is integrated directly to a chassis of electric motor with squirrel cage (fig. 1). In order to eliminate all the backlashes in the gearing and bearings, is branched in-to two identical gears (path A, path B ), between which is inserted the flexible component [1], [2]. The paths of transmission are located symmetrically around the the plane of symmetry of the motor. The rotor is equipped at the ends of eccentric cams on which are stored by rolling bearings toothed wheels (satellites). These are in mesh with the central wheels with internal teeth, which are rigid supported in the chassis of the motor. From the satellites is converted rotary motion to output shaft through an annular Oldham couplings and carriers.


Fig. 1. Internal structure of integrated gearbox: 1) stator with stator winding and ring gears, 2) squirrel cage with carriers, 3A) 3B) satellites, 4A), 4B) Oldham couplings, 5) output shaft.

## 3. BACKLASH ELLIMINATION

In order to elliminate all the the backlash in the gear mechanism must be assembled in a preloaded condition so that gears have been forced into mesh in opposite directions without external load. During operation, the power is transmitted to either one or the other branches of the transmission, depending on the direction of rotation of the input shaft (rotor). Each branch contains a separate arm of the output shaft. The preload is achieved by torsionally flexible connection between the both arms. The described prototype is designed such that the arms are mounted on a common continuous output shaft. So it is loaded with an additional torque $M_{T k}$, which is an optional parameter. However, since the output shaft is designed as a very stiff (in the described prototype it is a full steel shaft with diameter 12 mm and length 151 mm ), it is possible to achieve the function of backlash ellimination will to choose the value of the preload arbitrarily small. Because the assembly of the whole mechanism is statically indeterminate, the parameter $M_{T k}$ must be considered in each computation of force conditions.

The design of setting the torsion preload is solved by turning the ring gears to each other and fixing their positions in the motor chassis. In order to avoid any position change in the operation, it is necessary to use a form-fit joints. In this case, the selected connection pin. However, because the necessary angular position of gear rings continuously adjust, the set of pin holes is drilled around the all perimeter of ring wheels and motor chassis, and it works on the vernier principle. Moreover, the number of holes is bound by the number of teeth ring gears, so that the number of possible positions of gears is very high (see below). In engineering precision scales due to the fact the position is adjustable infinitely smooth (fig. 2).


Fig. 2. Vernier for the backlash ellimination in the transmission mechanism: 1) teeth number of ring gear $z_{k}, 2$ ) number of holes for the pin in ring gear $z_{k}+1,3$ ) number of holes for the pin in motor chassis $z_{k}+2$.

$$
\begin{equation*}
n=z_{k}\left(z_{k}+1\right)\left(z_{k}+2\right) \tag{1}
\end{equation*}
$$

Number of the possible angular positions of the ring gear $n$ is given by (1).

If the ring gear has 80 teeth for example (as described herein prototype), the total number of possible angular positions of installation is $80(80+1)(80+2)=531360$. This corresponds to an angular precision installation is approximately $\pm 2,5^{\prime \prime}$.

## 4. KINEMATIC SCHEME AND GEAR RATIO

Figure 3 shows a kinematic diagram of the gear mechanism. Each transfer branch is composed of members which presents table 1.

Furthermore, the table shows the kinematic constraints between individual members and the types of movement that they perform. Since all members perform plane motion in mutually parallel planes, the mechanism is solved as planar.


Fig. 3. Kinematic diagram of the gear integrated with electric motor.

| mechanism member |  | motion class |
| :---: | :---: | :---: |
| 1 | stator | fixed |
| 2 | rotor with carriers | rotary |
| 3 | satellite | planar |
| 4 | Oldham coupling | planar |
| 5 | output shaft | rotary |$\quad$| constraint | between members |
| :---: | :---: |
| rotary | $1-5,2-3,2-5$ |
| lincar | $3-4,4-5$ |
| general | $1-3$ |
| 8 |  |

Table 1. Members of the gear mechanism and the constraints between them.

The number of degrees of freedom for each branchis given by equation (2).

$$
\begin{equation*}
i^{\circ}=3(n-1)-2(r+p)-o=3(5-1)-2(2+2)-1=1^{\circ} \tag{2}
\end{equation*}
$$

Because the motor's rotor and output shaft arms are joined to an one kinematic member, and the ring gears are rigidly joined to motor chassis, the number of degrees of freedom for all mechanism is also $1^{\circ}$.

The gear ratio, derived by Willis method, is given by (3). In the equation (3) label $\omega_{2}$ is angular speed of the motor's cage, $\omega_{5}$ angular speed of the output shaft, zs teeth number of the satellite and $z_{k}$ teeth number of the ring gear.

$$
\begin{equation*}
i=\frac{\omega_{2}}{\omega_{5}}=-\frac{z_{s}}{z_{k}-z_{s}} \tag{3}
\end{equation*}
$$

## 5. DETERMINATION OF KINEMATICS USING MATRIX METHODS

### 5.1 Coordinate systems

The transmission mechanism is a planar system of bodies with one degree of freedom. To investigate the kinematic dependences must be appropriately chosen coordinate systems - global, which will be resolved to the movements of all members of the mechanism, and a local, strongly associated with moving bodies (figure 4).

At kinematic solution are all members considered as a rigid bodies. The arms of the output shaft of individual transmission branches are interconnected by an elastic element. Since other elements with which they are kinematically joined, are rigid, the arms are in operation relative to each other stationary. Consequently use of common coordinate system 5 for the output shaft and the arms.


Fig. 4. Coordinate systems: 1 - stator with ring gears, global coordinate system $O_{1} x_{1} y_{1} z_{1}$, fixed, 2 - rotor with carriers, coordinate system $O_{2} x_{2} y_{2} z_{2}, 3 \mathrm{~A}$ - satellite A, coordinate system $O_{3 A} x_{3 A} y_{3 A} z_{3 A}$, 3B - satellite B, coordinate system $O_{3 B} x_{3 B} y_{3 B} z_{3 B}, 4 \mathrm{~A}$ - Oldham coupling A, coordinate system $O_{4 A} x_{4 A} y_{4 A} Z_{4 A}, 4 \mathrm{~B}$ - Oldham coupling B, coordinate system $O_{4 B} X_{4 B} y_{4 B} Z_{4 B}, 5$ - output shaft with arms, coordinate system $O_{5} x_{5} y_{5} z_{5}$.

Kinematic variables of individual members and radiusvectors of the investigated points have upper index denotes the coordinate system in which these variables are related. If it is a variable in the global coordinate system, the upper index is omitted. (Example: coordinates of point $\mathbf{O}_{3 A}^{3 A}$ are related to coordinate systems 3 A , coordinates $\mathbf{O}_{3 A}$ are related to the global coordinate system.)

### 5.2 Coordinate transformation

To be able to solve variables in the global coordinate system, which necessary to determine the forces acting on individual members, it is necessary to construct the transformation matrix for the transition between different coordinate systems. Figures 5 to 12 show the relative position of coordinate systems and the corresponding transformation matrices.


Fig. 5. Transformation matrix $2 \rightarrow 1$


Fig. 6. Transformation matrix $3 \mathrm{~A} \rightarrow 2$


Fig. 7. Transformation matrix $3 \mathrm{~B} \rightarrow 2$


Fig. 8. Transformation matrix 4A $\rightarrow 3 \mathrm{~A}$

$\mathbf{T}_{4 \mathrm{~B} 3 \mathrm{~B}}=\left[\begin{array}{cccc}1 & 0 & 0 & x_{O 4 B 3 B} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$

Fig. 9. Transformation matrix 4B $\rightarrow 3 \mathrm{~B}$


$$
\mathbf{T}_{4 A 5}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & y_{O 4 A 5} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Fig. 10. Transformation matrix 4A $\rightarrow 5$


Fig. 11. Transformation matrix 4B $\rightarrow 5$


Fig. 12. Transformation matrix $5 \rightarrow 1$
The matrix index determines the direction of the transformation. (Example: transformation matrix $\mathbf{T}_{4 A 3 A}$ determines transformation $4 \mathrm{~A} \rightarrow 3 \mathrm{~A}$, transformation matrix $\mathrm{T}_{3 \text { A4A }}$ determines transformation $3 \mathrm{~A} \rightarrow 4 \mathrm{~A}$.) Matrices are extended, matrices dimension is $4 \times 4$. Matrix components at positions [1,4], [2,4] a [3,4] correspond to moving the origin of thecoordinate system in the dirrections $x, y$ a $z$. Transformed radiusvectors must therefore be a 4-dimensional
too. Their term is $r=[x, y, z]^{T}$. Because the investigated mechanism is a planar, the coordinate $z$ is zero. Transformation 2 $\rightarrow 1$ has general relation

$$
\begin{equation*}
\mathbf{r}^{1}=\mathbf{T}_{21} \mathbf{r}^{2} \tag{4}
\end{equation*}
$$

### 5.3 The kinematic dependences

As the independent variable of all movements was chosen the rotation angle of the rotor with carriers to the stator $\varphi_{2}$. This angle can be any function of time $\varphi_{2}=\varphi_{2}(t)$. From the kinematic constraints result dependences

$$
\begin{gather*}
\varphi_{5}=\varphi_{3 A}=\varphi_{3 B}=\varphi_{4 A}=\varphi_{4 B}=-\frac{1}{i} \varphi_{2},  \tag{5}\\
\varphi_{3 A}^{2}=\varphi_{3 B}^{2}=\varphi_{5}-\varphi_{2}, \tag{6}
\end{gather*}
$$

where $i$ is gear ratio (equation (3)). The equation (5) fully determines the motion of the output shaft. It remains to solve the general planar motion of the satellites and Oldham couplings.

Satellite A: Using basic decomposition of motion it is possible to decomposite the satellite A motion to a circular motion and a relative rotary motion. Reference point is $O_{3 A}$. Trajectory of the point is circular with radius $e$. The circular motion is given by

$$
\begin{equation*}
\mathbf{O}_{3 A}=\mathbf{T}_{21} \mathbf{T}_{3 A 2} \mathbf{O}_{3 A}^{3 A} . \tag{7}
\end{equation*}
$$

The equation (7) in a matrix form relates to equation (8).

$$
\left[\begin{array}{c}
O_{3 A x}  \tag{8}\\
O_{3 A y} \\
0 \\
1
\end{array}\right]=\left[\begin{array}{cccc}
\cos \varphi_{2} & -\sin \varphi_{2} & 0 & 0 \\
\sin \varphi_{2} & \cos \varphi_{2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
\cos \varphi_{3 A}^{2} & -\sin \varphi_{3 A}^{2} & 0 & 0 \\
\sin \varphi_{3 A}^{2} & \cos \varphi_{3 A}^{2} & 0 & e \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
0 \\
0 \\
0 \\
1
\end{array}\right]
$$

Substitute equations (5) and (6) and expand (7) to components gives equations for coordinates of point $O_{3 A}$ in coordinate system 1:

$$
\begin{gather*}
O_{3 A x}=-e \sin \varphi_{2},  \tag{9}\\
O_{3 A y}=e \cos \varphi_{2} . \tag{10}
\end{gather*}
$$

Relative rotary motion is given by (5). The motion description of satellite B is analogous.

Oldham coupling A: Reference point of basic decomposition is the point $O_{4 A}^{4 A}=[0,0,0,1]^{T}$. Moving motion is given by

$$
\begin{equation*}
\mathbf{O}_{4 A}=\mathbf{T}_{21} \mathbf{T}_{3 A 2} \mathbf{T}_{4 A 3 A} \mathbf{O}_{4 A}^{4 A} . \tag{11}
\end{equation*}
$$

By multiplication of equations (11) and substitution of (5) and (6) gives the equations for moving motion

$$
\begin{gather*}
O_{4 A x}=x_{O 4 A 3 A} \cos \frac{\varphi_{2}}{i}-e \sin \varphi_{2}  \tag{12}\\
O_{4 A y}=-x_{O 4 A 3 A} \sin \frac{\varphi_{2}}{i}+e \cos \varphi_{2} \tag{13}
\end{gather*}
$$

With the kinematic constraints must also apply the equation

$$
\begin{equation*}
\mathbf{O}_{4 A}=\mathbf{T}_{51} \mathbf{T}_{4 A 5} \mathbf{O}_{4 A}^{4 A} . \tag{14}
\end{equation*}
$$

Component equations for position of point $O_{4 A}$ in coordinate system 1 are

$$
\begin{align*}
& O_{4 A x}=y_{O 4 A 5} \sin \frac{\varphi_{2}}{i},  \tag{15}\\
& O_{4 A y}=y_{O 4 A 5} \cos \frac{\varphi_{2}}{i} . \tag{16}
\end{align*}
$$

From a comparison of the right-hand sides of equations (12), (13), (15) and (16) result a system of two linear equations with a parameter $\varphi_{2}$ for unknown displacement of coordinate systems $x_{\text {O4A3A }}=x_{\text {OAABA }}\left(\varphi_{2}\right)$ and $y_{O 4 A 5}=y_{\text {OAA5 }}\left(\varphi_{2}\right)$. In matrix notation the system has the form

$$
\left[\begin{array}{cc}
\cos \frac{\varphi_{2}}{i} & -\sin \frac{\varphi_{2}}{i}  \tag{17}\\
-\sin \frac{\varphi_{2}}{i} & -\cos \frac{\varphi_{2}}{i}
\end{array}\right]\left[\begin{array}{c}
x_{O 4 A 3 A} \\
y_{O 4 A 5}
\end{array}\right]=e\left[\begin{array}{c}
\sin \varphi_{2} \\
-\cos \varphi_{2}
\end{array}\right] .
$$

Because the system matrix is symmetric and orthogonal, apply to an unknown displacement relation

$$
\left[\begin{array}{c}
x_{O 4 A 3 A}  \tag{18}\\
y_{O 4 A 5}
\end{array}\right]=e\left[\begin{array}{cc}
\cos \frac{\varphi_{2}}{i} & -\sin \frac{\varphi_{2}}{i} \\
-\sin \frac{\varphi_{2}}{i} & -\cos \frac{\varphi_{2}}{i}
\end{array}\right]\left[\begin{array}{c}
\sin \varphi_{2} \\
-\cos \varphi_{2}
\end{array}\right] .
$$

Expand (18) to components and modification gives

$$
\begin{align*}
& x_{O 4 A 3 A}=e \sin \left(\varphi_{2} \frac{1+i}{i}\right),  \tag{19}\\
& y_{O 4 A 5}=e \cos \left(\varphi_{2} \frac{1+i}{i}\right) . \tag{20}
\end{align*}
$$

Substitute (20) to (15) and (16) we get the final expression for the position of the reference point $O_{4 A}$ at global coordinate system 1 in the form

$$
\begin{align*}
& O_{4 A x}=e \cos \left(\varphi_{2} \frac{1+i}{i}\right) \sin \frac{\varphi_{2}}{i},  \tag{21}\\
& O_{4 A y}=e \cos \left(\varphi_{2} \frac{1+i}{i}\right) \cos \frac{\varphi_{2}}{i} . \tag{22}
\end{align*}
$$

Relative rotary motion is given by (5). The motion description of Oldham coupling B is analogous.

## 6. CONCLUSION

The aim of paper was to introduce and explore in detail the kinematics of special gear mechanism, which is integrated with the driving electric motor in one unit and thus form a rotary electric actuator. Detailed determination of kinematic variables dependency mechanism is needed for a specific engineering design the drive of that principle.

Further work will mainly cover the design of gear with respect to secondary interference on the heads of teeth and optimize the design to the strength.

Acknowledgements: The research work was made possible by VUTS, a.s. Liberec on research project FR-TI1/594.

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