# Deformation of the elastic wheel harmonic gearing and its effect on toothing

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**Abstract** Harmonic gears are lighter and smaller in comparison to regular toothed gears. It is characterized by a high kinematic accuracy, less noisy and has up to 5 times higher damping capacity than current transfers. A harmonic toothed gear is basically a differential gear with frontal gearing where the meshing is achieved by a flexible deformation of one of the wheels.

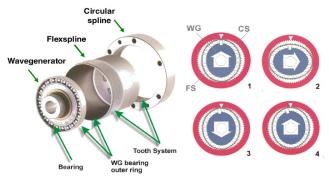
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3 - As the wave generator rotates 180 degrees clockwise, the flexspline moves counterclockwise by one tooth relative to the circular spline. 4 - Each full turn of the wave generator moves the flexspline two teeth in the opposite direction relative to the circular spline.

Fig.1 Principle of operation smooth transfer

#### 1. HARMONIC DRIVE

A harmonic gear is basically a differential gear with a train of spur gears where the mesh is achieved by the flexible deformation of one of the meshing wheels Flexible wheel 1 has external teeth; solid wheel 2 has internal gearing. Both bikes have the same module and diagonal. Flexible wheel has fewer teeth than the solid wheel. Under the effect of the wave generator the flexible wheel deforms and the teeth of the flexible wheel merge with the tooth gaps on the rigid wheel – they mesh (Fig. 1).



1 - The flexible is slightly smaller in diameter than the circular spline, resulting in its usually having two fewer on its outer circumference. It is held in an elliptical shape by the wave generator and is teeth engage with the circular spline across the major axis of the ellipse. 2 - As soon the wave generator starts to rotate clockwise, the zone of tooth engagement travels with the major elliptical axis.

# 2. THE DEFORMATION OF WHEEL AND IMPACT ON GEAR

The existence of the flexible wheel within the harmonic gear, which undergoes deformation during the process of usage, requires a specific approach in the mesh examination of this gear. In particular it is necessarily to give the influence of the deformation of the flexible wheel in the shape of the teeth thereon spaced.

Flexible wheel during operation straining very negative The following are the main stresses:

- Deformity Stress induced by the generator
- Stress induced/generated by the transmitted load
- Local stresses from the tooth flexion within the tooth gaps.

As a result of the adverse stress the flexible wheel is the limiting part of the load-bearing capacity of the harmonic gears.

Experimental results [4] had shown that the change in profile form and the tooth position is not significant in possible figures of the flexible wheel deformation and their real ratios between the average of the middle area (middle area is a geometrical area of points dividing the thickness of the flexible wheel wall into two) and of the thickness of the flexible wheel wall.

The flexible deformation of the harmonic wheel may be twofold:

a) The free deformation at which the harmonic wheel is deformed by means of rollers of a smaller cross-section (Fig. 2.a). Harmonic wheel is deformed to shape of ellipse.

b) Forced deformation, where the shape of the wheel is given by the shape of the generator harmonic deformation (Fig.2. b).

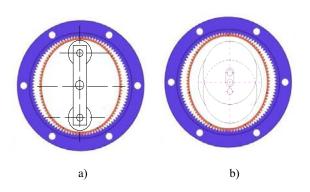


Fig. 2 Deformation of harmonic gear

- a) the loose deformation of harmonic wheel
- b) forced deformation of harmonic wheeô

The second case is of preferred, since the harmonic wheel can be deformed into any shape. If the evolving gear is to be used the spacing curve has to be in the circular shape in the meshing of the harmonic wheel after the deformation. The magnitude of the flexible wheel deformation has a considerable influence on the character and the quality of meshing. Let us consider this question regardless of the thickness of the flexible wheel and the gear load.

For friction gear is radial deformation  $w_o$  clearly intended conditions of contact wheel, according to which:

$$2.w_{o} = d_{v3} - d_{v2} \tag{1}$$

where:  $d_{v2}$  - outer diameter of the flexible wheel  $d_{v3}$  - inner diameter of the solid wheel

a gear ratio: 
$$t_{12}^3 = -\frac{d_{v2}}{d_{v3} - d_{v2}} = -\frac{d_{v2}}{2 \cdot w_a}$$
 (2)

Then in the geared transmission:

$$i_{12}^{3} = -\frac{Z_{2}}{Z_{3-}Z_{2}} = -\frac{Z_{2}}{K_{z} \cdot U}$$
(3)

Where:  $K_Z$  - coefficient difference in the number of teeth U - number of waves deformation

From the relationship (3) gear ratio does not depend on the size of radial deformation. This may be any, provided that there is secured generating.

From the relationship (3) follows :

$$\dot{I}_{12}^{3} = -\frac{Z_{2.m}}{/Z_{3} - Z_{2}/.m} = -\frac{d_{v2}}{d_{v3} - d_{v2}}$$
(4)

which is equal to ratio (2). Therefore, we use the equation (1) for geared transmission:

$$W_o = /z_3 - z_2 / .m \Rightarrow W_o = \frac{m . / z_3 - z_2 /}{2} = \frac{m . K_z . U}{2}$$
 (5)

For the size of deformation double wave gearing, where U = 2, Kz = 1, then:

$$W_0 = m \tag{6}$$

On (Fig. 3) there are trajectories of the movement of flexible wheel teeth in regard to the teeth of the rigid wheel for three examined alternatives [3]. The course of the tooth entering meshing is illustrated in a solid line. Points A can be considered as the central ones / i.e. the points of the beginning of the meshing / in an unloaded gear and approximately also in the loaded one, too.

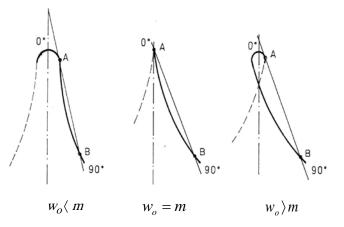


Fig. 3 Trajectories of the movement of flexible wheel teeth in regard to teeth of the rigid wheel

The most effective meshing zone is the zone of the major axis of the generator, where  $\phi = 0$  as that produces the maximal meshing and the minimal slipping. Because of this is  $w_0 = m$  very effective. If  $w_0 < m$ , the meshing zone moves in the direction of the generator's movement.

In the zone of the generator's major axis the meshing slipping increases, which has a decreasing influence on the efficiency and the increasing influence on wear. The usage of  $w_0 < m$  is justified in a situation when it is necessary to decrease the bending stress in the flexible wheel. In the case of  $w_0 > m$  the meshing zone moves against the direction of the generator's movement. Again the slipping occurs and the beginning of the meshing can be located on the curve of the tooth entering the meshing. The magnitude of the deformation  $w_0 > m$  is used with large gear ratios to compensate for the deformation due to a considerable load.

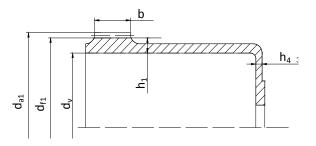
### 3. REQUIREMENTS FOR TOOTH PROFILES

Curve tooth flanks shall be so designed to ensure that:

- 1. The current gearing greater number of teeth
- 2. Correct meshing conditions
- 3. The cheap production and the like.

These conditions that are best for profile of the tooth flanks have involute curve, which is advantageous in terms of production technology, the provide a suitable tooth flanks. Harmonic transfers have a large number of teeth. In case the number of teeth z > 150involute is coming to the straight line. Depth gearing is limited to angle the tool  $\alpha = 20^{\circ}$  and  $h_0 = m$ . Flanks tooth a flexible wheel must be envelope curve the tooth flanks of the solid wheel. The condition of using the evolving profile is that the rolling curve of the flexible wheel would be circular in the engaging part.

### 4. BASIC DIMENSIONS OF FLEXIBLE WHEEL





The dimensions of the flexible wheel according to (Fig. 4)

$z_1 = 172$	$d_{f1} = 102,4 \text{ mm}$	b = 16 mm
$h_4 = 0.9 \text{ mm}$	$d_{a1} = 104,02 \text{ mm}$	
$h_1 = 1,2 \text{ mm}$	$d_1 = 97,85 \text{ mm}$	

The type of chosen generator cam is made in the shape of  $\cos 2^{\varphi} /\approx$  elliptical /. Because the cam and resilient wheel is standard special single row ball bearings with flexible ring type LGVZ, I choose the internal diameter of the flexible wheel d<sub>v</sub> = 100 (mm.)

#### 5. DETERMINATION OF THE WHEEL DEFORMATION USING FEM

The flexible wheel is under very disadvantageous stress during the operation. There is stress from the generator deformation, stress from transmitted load and local stress from the tooth flexion within the tooth gaps.

As mentioned above, the extent of deformation of the elastic wheel has a significant impact on the character and quality of the image. The issue is devoted to deformation of the teeth lot of work. In the experiment conditions the tooth deformation is mostly determined by a static measurement of the tooth deformation loaded with a constant power or it is determined with the measurement of the divergence during a slow rotation. Currently, to address these issues using modern methods, which include the finite elements method -FEM, which is one of the widely used numerical methods.

Size of the deformation of the elastic wheel is detected in two cases, namely:

Alternative 1 - if the force is emerging from the wave generator so it is applied in the centre of the tooth (Fig. 5 a).

Alternative 2 - if the force is emerging from the wave generator so it will operate in the middle of tooth gap (Fig. 5 b).

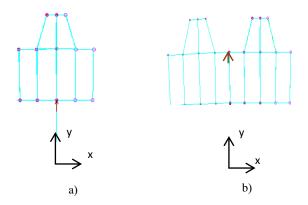


Fig. 5 a) saving the applied force in the middle tooth, b) saving the applied force in the a notched gap

After performing a static calculation and using the command STRPLOT the determine size of emerging tensions in the deformed wheel (Fig. 6).

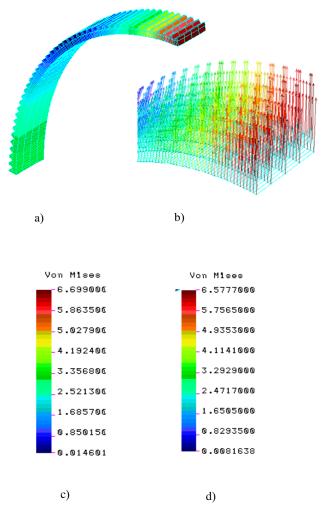


Fig. 6: a) the deformed shape of the flexible wheel, b) vector rendering of deformation of the wheel, c) colour resolution of the size voltage and the values voltage if the force is applied in the middle tooth, d) colour resolution of the size voltage and the values voltage if the force is applied in a notched gap.

When comparing the value of emerging tensions in the event that force is applied in the middle of the tooth or if force is applied in the tooth clear space find that if force is applied in the middle tooth voltage values are higher than in the case where the force is applied in the middle tooth gap.

After determining the shape of the deformed tooth it is necessary to design an appropriate shape of the opposite profile so when meshing the flexible wheel with the rigid wheel of the harmonic gear it would not cause interference. Tooth flanks solid wheel must be enveloping curves of the tooth flanks of the flexible wheel. The internal gear is when the outer and the inner teeth mesh together. The harmonic gear is such a case where the outer teeth are provided by the flexible wheel and the inner by the rigid wheel. During meshing the associated teeth profiles are in point contact at all times. Associated profiles that ensure a constant gear ratio we can determine:

- 1. direct manner
- 2. indirect manner

A direct manner the creating gear is such that one profile is selected and the other is derived from it. In indirect manner is chosen point or profile other auxiliary wheel that by rolling the tooth profiles of the two wheels defines the associated profiles.

#### 6. CONCLUSION

The harmonic gearing unquestionably include among a prospective technology. A harmonic gear is basically a differential gear with a train of spur gears where the mesh is achieved by the flexible deformation of one of the meshing wheels. The extent of the flexible wheel deformation is coherent to the character and the mesh quality. The difference lies in the fact that more cogs/teeth participate in meshing and thus also in transmission at the same time. The existence of the flexible wheel within the harmonic gear, which undergoes deformation during the process of usage, requires a specific approach in the mesh examination of this gear. Processing of computed results is an important part of the final calculation by finite element method. Processing of the calculated results is important, the final part of the calculation by finite element method. The tasks of the mechanics of deformed bodies are generally the most important results of the nodal displacements, stress and deformation.

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