Impact Analysis of the Geometric Parameters of the Spur Gears on the Teeth’s Stiffness

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Abstrakt Deformation of teeth is usually expressed quantitatively by stiffness gearing. One of the factors that aggravate the environment is a noise. The noise in gear transmissions particularly affects periodic change of teeth stiffness during meshing, caused by changing the number of pairs of teeth, which are simultaneously in meshing. Tooth stiffness is depends on the shape of the teeth, on the basic parameters of examined spur gear, such as the number of teeth, module gearing, pressure angle, gearing width, correction and modification of gearing. The article is devoted to the problems of gearing stiffness analysis. The problem is solved for spur gears. Deformation analysis solved by FEM is used for calculations of the gearing stiffness.

Key words spur gear, teeth stiffness, FEM, teeth deformation

1. INTRODUCTION

Gearing is a phenomenon of our culture as well as his wheel and circular motion. The history of gears is probably as old as civilization itself. The earliest description of gears was written in the 4th century B.C. by Aristotle. He wrote that “direction of rotation is reversed when one gear wheel drives another gear wheel”.

The internal dynamics of the teeth is one of the most common gearing problems. This is reflected in vibrations of all gear parts, their noise and increased teeth stress [4]. The noise in gear transmissions particularly affects periodic change of teeth stiffness during meshing, caused by changing the number of pairs of teeth, which are simultaneously in meshing. One of the ways to specify the teeth stiffness is calculated using the total deformation gearing.

This problem is addressed in many works. The older works are based on the classical theory of elasticity and tooth of gear is considered as cantilevered beam. The existing experimental techniques are based on static deflection measurements gearing loaded of constant force or seismic measurement deviations at slow rotation. Recently, we meet with computer-aided design methods for gearing in the available literature. These modern methods also include the finite elements method - FEM, which is one of numerical methods in mathematics [4]. Using FEM we can to solve the direct deformation of teeth of spur gear with sufficient accuracy.

Creation of a geometric model of the gear is considered the first step to deal with tooth deformation FEM. There is no universal way in which geometry-computer model can be created. An effective procedure is to transfer the geometry from any CAD system (such as AutoCAD, Bentley, ProEngineer, I-DEAS, Solid Works, etc.) [8]. To determine the computer model for the studies of deformation of the teeth using FEM was necessary to determine material constants, define the type of the finite element, and to select appropriate boundary conditions (geometry and power).

2. DEFINITION OF TEETH STIFFNESS OF SPUR GEAR

Gear teeth are deformed due to the load. Deformation of teeth is usually expressed quantitatively by stiffness gearing. Periodic changes in the stiffness tooth mesh, caused by changes in the number of pairs of teeth, which are also mesh in a significant noise impact on teeth. One of the ways to specify the tooth stiffness is calculated using the total deformation gearing.

In general the resulting stiffness c defined by equation (1):

$$c = \frac{W}{\delta} = \sum_{p} c_{p}, \quad p = I, II \quad [N/mm, \mu m]$$

where

- $w$ - load across the width of the teeth $[N/mm]$,
- $w_I$ - load across the width of the first pair of teeth, $w_{II}$ - load across the width of the second pair of teeth,
\[ \delta = \text{resulting deformation} \ [\mu \text{m}] \].

The resulting stiffness the teeth equal to the sum of partial stiffness of pairs of teeth, which are mesh. The stiffness of each pair of teeth is calculated according to equation (2) to final stiffness of a pair of teeth:

\[ \frac{1}{c_p} = \frac{1}{c_1} + \frac{1}{c_2} \]  

(2)

where

- \( c_p \) - resulting stiffness of a pair of teeth [N/mm.\mu m],
- \( c_{1/2} \) - stiffness of each tooth to which it applies

\[ c_{1/2} = \frac{w}{\delta_{1/2}} \ [\text{N/mm.}\mu \text{m}] \].

The stiffness is individual pairs of teeth in the mesh by changing the length of the engaging line. The minimum value shall end in the engaging points and lines shall at maximum point lone mesh, the so-called pitch point C. The resulting stiffness teeth after track mesh changes periodically with a period equal to the basic pitch frontal. The endpoints solitary mesh leads to sudden changes in stiffness resulting teeth. This is due to a step change in deformation resulting from the entry into another pair of teeth in the mesh his cause's vibrations that cause noise gearbox.

3. INFLUENCE OF PARAMETERS ON TEETH STIFFNESS

The teeth stiffness of meshing of spur gears is not constant for all examined teeth of gears. The stiffness of the teeth depends on the shape of the teeth, thus the basic parameters of the gearing, such as the number of teeth, module gearing, pressure angle, gearing width, correction and modification of gearing and on the shape and construction of the body wheel and the wheel load. The value of teeth stiffness is solution by deformation of teeth. The deformation of teeth is solution by finite element method.

3.1 Influence of the number of teeth on the stiffness of teeth

The basic parameter of spur gear is the number of teeth. Figure 1 shows the influence of the number of teeth on the meshing stiffness of teeth. This impact is studied on the theoretical one pair meshing. The number of teeth of examined spur gears are 17, 19, 21, 27, 35 and 61, the teeth module is \( m_n = 1 \text{mm} \), the width of the gear wheels is \( b = 20 \text{ mm} \) and load is \( F = 1000 \text{N} \). The stiffness of the teeth are examined at the meshing contact for the characteristic points of meshing using finite element method. With an increased number of teeth, but if the other parameters of gearing are non-change, increases the teeth stiffness of spur gear.

The influence of the number of teeth on the stiffness is also shown in Figure 2. It is designed for the same parameters as in Figure 1.

![Fig.2 Influence of number of teeth “z” on the tooth stiffness](image)

3.2 Influence of the tooth module on the stiffness of teeth

This is known as the module or modulus of the wheel is simply defined as share the circular pitch to the number \( \pi \). For solving the problem using the finite element method is an important accurate selection of finite element method mesh dimension at each model examined gears. Figure 3 shows the course of the stiffness in the place of load, if the force is applied to the top of tooth. The problem is resolved on the models of spur gears with the number of teeth \( z = 19 \), the face width \( b = 20 \text{ mm} \), the load force is \( F = 500 \text{ N} \) and module is be changing.

The results show that the stiffness of the teeth is not depending on the size of the teeth - on the module.

![Fig.3 Influence of module on the tooth stiffness](image)

3.3 Influence of the length of meshing contact on the stiffness of teeth

In practice, we encounter cases where the width of load is less than the width of gearing. Therefore, let us consider the case where the width of gearing remains unchanged (\( b=20\text{mm} \)) and the width of load is reduced by 0,5 mm on each side (Fig. 4 - a) in the first case, and about 2,5 mm in the second case (Fig. 4 - b).

![Fig.4 Influence of number of teeth on the tooth stiffness in the meshing points](image)
Figure 5 the course of the stiffness of the teeth for figure 4-a). It is for model of spur gear with number of teeth \( z=19 \), the module of teeth \( m=1 \text{mm} \), the load \( w=26,3179 \text{N/mm} \) (\( F=500 \text{N} \)), width of gearing \( b=20 \text{mm} \) and width of load \( b_w=19 \text{mm} \).

Figure 6 the course of the stiffness of the teeth for figure 4-b). It is for model of spur gear with number of teeth \( z=19 \), the module of teeth \( m=1 \text{mm} \), the load \( w=33,33 \text{N/mm} \) (\( F=500 \text{N} \)), width of gearing \( b=20 \text{mm} \) and width of load \( b_w=15 \text{mm} \).

Resulting from Fig. 5 and Fig. 6 that if the width of load is less than the width of gearing, the stiffness in the edges of the meshing has a sharp increase.

The teeth of gearing are deformed due to the load. Recently, at ever faster evolving computer technology and the available literature, we can encounter modern numerical methods, such as finite element method, which can serve as methods for the determination of deflection gearing. The deformation of pairs of teeth over the meshing along the line of action is changing and this value is influenced of parameters of spur gears. The change of teeth deformation causes the change of meshing stiffness. This is the result of vibrations of noise in the gearbox.

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References