GRANT journal
ISSN 1805-062X, 1805-0638 (online) Vol. 11, issue 02
EUROPEAN GRANT PROJECTS | RESULTS | RESEARCH & DEVELOPMENT | SCIENCE

Simulation and mathematical modeling of table-tenis ball

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Grant: VEGA 1/0201/21
Název grantu: Mobile mechatronics assistant
Oborové zaměření: BC - Teorie a systémy řízení

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Abstrakt The article deals with the creation of the mathematical and dynamic model of a table-tenis ball. There is the basic theoretical concepts related to the problem of creating mathematical models. The basic element of the simulation was the solution of the equation of motion for table-tennis balls. The equation of motion needs a mathematical description of the acting forces. The basic equations of motion necessary for further solution of the paper are derived. The is devoted to the creation of the actual dynamic model of the ball using the dynamic equations of motion. Matlab/Simulink is used for the modelling, which allows for easy data processing afterwards.

Klíčová slova table-tenis ball, simulation model, modeling

1. INTRODUCTION

Mathematical modeling has become an effective method for solvingscientific and technical problems. A computer model constructed on the basis of mathematical analysis represents a surrogate sample of the object under study. With the model we can perform experiments that are similar to the experiment on the real object. The great advantage of mathematical modelling is its possibility of infinite repetition without damaging the object. This process is called simulation.
Thanks to the rapid development of simulation programs, it is possible to perform increasingly complex calculations in various scientific directions. Using mathematical modelling, it is possible, for example, to predict climate change, the economic state of a country, as well as the behaviour of a four-stroke engine under a certain state of stress.

1. Mathematical model of table-tenis ball

By a mathematical model we mean an abstract model that uses mathematical notation to describe the behaviour of a system. A mathematical model transforms the model into mathematical notation and has the following advantages:

- formalization of the notation given by historical development,
- precise rules for manipulating mathematical symbols,
- the possibility of using information and communication technologies to process the model created.

It is not possible to describe real systems, objects or processes that are very complex by mathematical notation, therefore the most important parts of the system to be modelled have to be identified first.

The basic components of a mathematical model are:

- variables and constants,
- mathematical structures,
- solutions.

1.1 Basic types of mathematical models

In the course of identification and analysis of the system modelling, it is useful to determine to which category the mathematical model belongs, which will make it easier to recognize the basic properties and structure of the model being sought. Based on the general classification of models, models are divided into:

- deterministic and stochastic models, the nature of the assumed relationships between the variables that characterize the system under study. Deterministic variables are known quantities that can be accurately measured and controlled. Stochastic variables are unknown variables that cannot be measured and controlled accurately because they are random in nature. Deterministic models assume certain deterministic - functional relationships between variables (a certain value of one independent variable is associated with a certain value of the other dependent variable). The model is constructed on the basis of basic theoretical laws (thermodynamic, hydrodynamic laws, etc.). Stochastic models are random relationships between variables (a certain value of one variable corresponds to different values of the dependent variable). The model is based on the processing of experimental results.
- dynamic and static models, dynamic and static models are divided according to the explicit data on the evolution of the system over time. They represent a description of the coupling

over time, in the transition from one state to another, and can be

- active (controlled) and passive (uncontrolled). A static model is a model that describes the linkages between the fundamental variables of a steady-state process. Since dynamic models are complex, they are often replaced by a sequence of simpler static models.
- models with concentric and distributed parameters, concentric and distributed parameter models are divided based on the properties of the variables. We distinguish between models with distributed and with concentric parameters. Decomposed parameter models, where the basic variables vary in both time and space (expressing the solution in terms of partial differential equations, integral relations). In Concentric Parameter Models, either time or location in space varies (expressing the solution using classical differential equations).
- continuous and discrete-time models, Continuous and discrete-time models are divided into basic behaviour of the underlying variables. In the Continuous model, the variables are determined at any point in time. They are used in the study of systems that involve continuous processes that are described by differential equations expressing the rate of change of variables with time. A discrete model acquires variables only at a certain point in time.

2. CREATE A MATHEMATICAL MODEL

The ball is assumed to fall from height h0 onto a table that has a smooth, horizontal surface. The ball is modeled as a mass point and therefore the rotation of the ball is neglected. Vertical motion is considered only as schematically depicted in Fig. 1, which indicates the position of the ball from the surface as a function of the time of first touches. Due to the inelastic nature of the contact between the ball and the table, the maximum height of the ball decreases gradually with each impact.

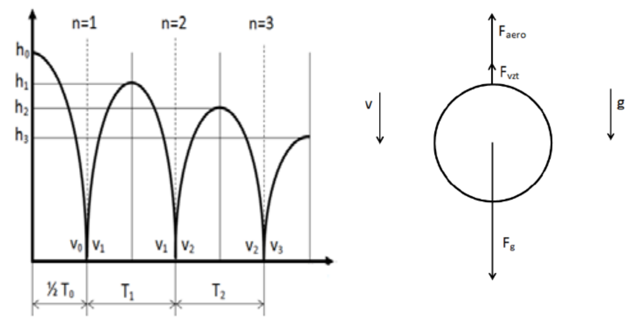

Fig. 1 Vertical movement and relase of the ball

The course of the ball's impact, shown in Fig. 1, calculates three external forces:

- gravitational force of the ball:
  $F = m.g$ (1)

- lifting force:
  $F_l = \rho.g.V$ (2)

where

$$\boldsymbol{V = \frac{1}{6}\pi.D^3}$$ (3)

- aerodynamic resistance force:

$$F_a = \frac{1}{2}\rho.A.C_D v^2$$ (4)

where
m – mass
v – speed
A - cross-sectional area
V - ball volume
D - ball diameter
$\boldsymbol{\rho}$ - air density
CD - drag coefficient
g - gravitational acceleration

As the ball moves upwards, the aerodynamic force in Fig. 1 acts in the opposite direction. For a ping pong ball in air at room temperature, it is shown that the buoyant force Fl is approximately one percent of the mass. Therefore, buoyancy is assumed to have a negligible effect.

Movement equation for the falling ball is:

$$g = \frac{dv}{dt} = \ddot{y} \quad (5)$$

Movement equation for the rising ball is:

$$-g = \frac{dv}{dt} = \ddot{y} \quad (6)$$

Assuming that air resistance is neglected, the flight time between bounces consists of equal and symmetric rise and fall time phases, i.e., the time from the bounce of the ball to reaching the top of the trajectory is equal to the time it takes to get from that top back to the surface of the table, of which each is half of the total flight time Tn between the nth and (n+1) bounce. The velocity after impact vn, which is the velocity associated with the nth bounce, is then:

$$v_n = g\left(\frac{T_n}{2}\right) \qquad n = (1,2,3 \dots.) \quad (7)$$

The relationship for the total flight time Tn is:

$$T_n = e^n\left(\frac{2v_o}{g}\right) \qquad n = (1,2,3 \dots.) \quad (8)$$

where v_o is initial speed.

3. SIMULATION MODEL

To create the mathematical model, we will start from the equation of motion for the falling ball, which has the shape:

$$-g = \frac{dv}{dt} = \ddot{y}$$

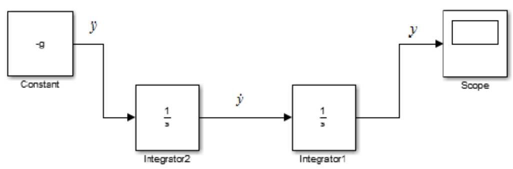

Fig. 2 Simulation model

The dependence of the height on time is a parabola, because after double integration of the equation of motion we get the relation:

$$y(t) = y(0) - \frac{1}{2}gt^2 \tag{9}$$

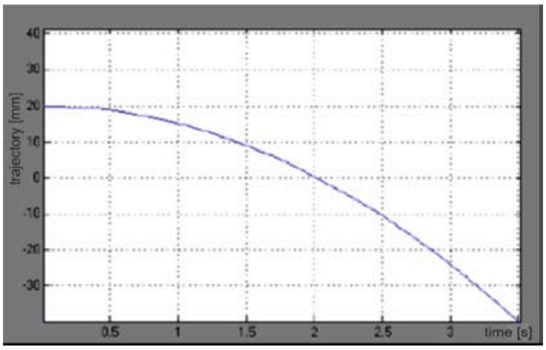

Fig. 3 Trajectory of table-tenis ball

There are several options for implementing the bounce of the ball off the table. One possibility is to multiply the ball velocity $\dot{y}(t)$with a coefficient 1< k < 1 at the moment the ball hits the table. Thus, the ball starts to will start moving in the opposite direction (upwards) and the ball speed will gradually decrease due to the bounces.

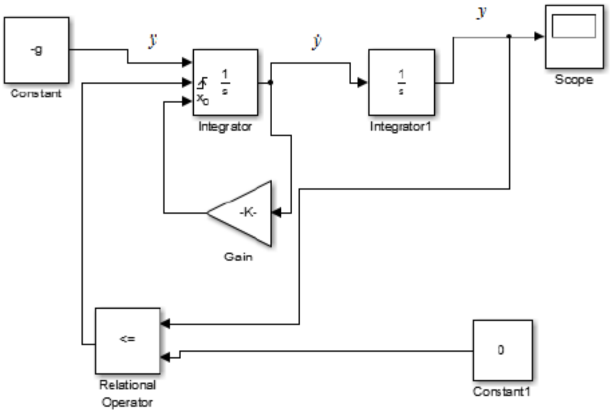

Fig. 4 Simulation model with algebraic loop removed

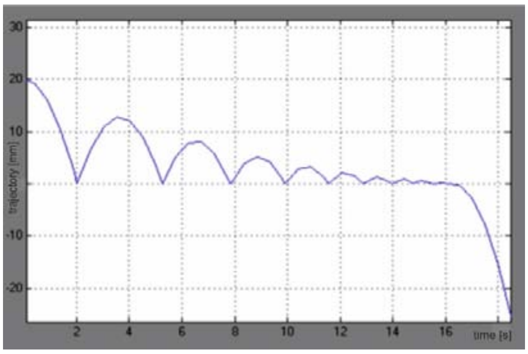

Fig. 5 Bouncing the ball

The graph in Fig. 5 shows that the ball is indeed already bouncing, decreasing in height and velocity with each impact. However, the graph shows an error in the area where the ball should settle. Simulink solves the differential equations in certain time steps, and the time between two bounces is approximately equal to the length of the simulation step. At such a time, the value at the output of the comparison operator does not change, there is no second reset of the integrator, and the ball falls down. There are several ways to solve this problem, but probably the simplest way is to set a minimum value constraint on the output of the right integrator.

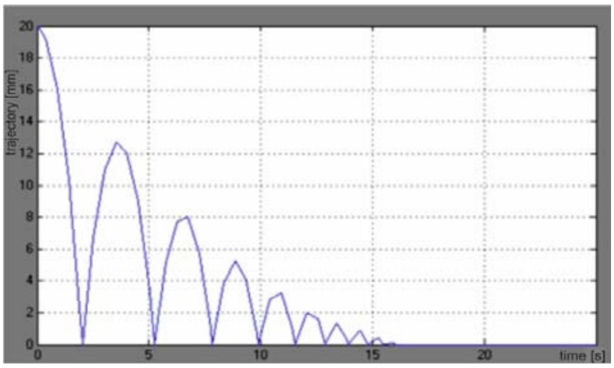

Fig. 6 Bouncing the ball off the table

4. CONCLUSION

The aim of the paper was to create a mathematical model of a ping pong ball. The first part describes the basic concepts related to the problem. In the second part, since the ball behaves as a material point, we had to define the kinematics and dynamics of the material point. Once defined, we could start from the formation of the dynamic equations of motion. For simplicity, it was assumed that the ball falls from a certain height onto a horizontal pad, from which it bounces until it comes to rest. Next, we discussed the creation of a computer model in a simulation program. We created the model using schematics that, when properly connected, plotted the velocity and trajectory of the ball. To create the mathematical model we used Matlab/Simulink software.

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