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Modeling in transport and distribution logistics

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Abstrakt Logistics for distribution and transportation are essential to the effective flow of products and services globally. Modeling approaches in transportation and distribution logistics have become crucial tools for streamlining operations, cutting expenses, and guaranteeing on-time delivery as supply chains grow more intricate and worldwide. The theoretical underpinnings of transportation and distribution logistics modeling are examined in this article, along with real-world examples that demonstrate how to use it.

Klíčová slova distribution logistic, network, routing optimization

Transport and distribution logistics form the backbone of modern supply chains, ensuring goods are delivered to customers efficiently and cost-effectively. With increasing complexity in global trade, robust and accurate modeling techniques have become indispensable for optimizing these operations. Modeling serves as a strategic tool for decision-makers, enabling them to design, analyze, and improve logistics networks. This article explores the theoretical underpinnings of transport and distribution logistics modeling and examines its applications through real-world examples.

1. THEORETICAL FOUNDATIONS

Managing the movement of resources, information, and items from the point of origin to the site of consumption is the focus of transport and distribution logistics. The objective is to accomplish this as cheaply and quickly as possible while taking environmental sustainability into account.

1.1 Definitions and Key Concepts

Transport and distribution logistics involve the movement of goods, materials, or information from origin points to consumption points. Modeling serves as an abstraction of real-world logistics systems, allowing analysts to capture essential features and solve problems efficiently. These models are indispensable tools for:

- *Network design*: Determining optimal locations for warehouses and distribution centers.
- *Fleet management*: Allocating vehicles and routes to meet demand with minimal costs. [7], [9]
- *Last-mile delivery optimization*: Addressing challenges in urban logistics to minimize delays and improve customer satisfaction.

A robust logistics model incorporates three primary dimensions:

- *Spatial*: The geographical layout, including transportation networks, facilities, and customers.
- *Temporal*: The timing of activities, including production, storage, and delivery.
- *Functional*: The interdependence between logistics functions, such as procurement, inventory, and distribution.

1.2 Types of Models

Logistics models [7] can be classified based on their mathematical structure, problem domain, and the type of solution sought.

I. **Mathematical Models**
Mathematical models represent logistics systems using equations or inequalities. Examples include:

- *Linear Programming (LP)*: Solves optimization problems with linear relationships, e.g., minimizing costs while meeting customer demand.
- *Integer Programming (IP)*: Extends LP by incorporating decision variables that must take integer values, such as the number of vehicles.
- *Dynamic Programming (DP)*: Solves problems that evolve over time, such as inventory replenishment schedules.

II. **Simulation Models**
Simulation models use computational tools to mimic real-world processes. They are particularly useful for capturing system dynamics and uncertainty, such as traffic congestion or demand variability. Common methods include:

- *Discrete-Event Simulation (DES)*: Models the operation of logistics systems as a sequence of events, such as arrivals and departures at a warehouse.
- *Agent-Based Simulation (ABS)*: Represents individual agents (e.g., drivers, customers) to study interactions and emergent behaviors.

III. **Optimization Models**
Optimization models are designed to find the best possible solution to a logistics problem while satisfying constraints. These models often employ:

- *Exact Algorithms*: Provide guaranteed optimal solutions but may be computationally expensive (e.g., branch-and-bound, simplex method).
- *Heuristic Algorithms*: Generate good solutions quickly but without a guarantee of optimality (e.g., nearest neighbor, Clarke-Wright savings). [2], [16]
- *Metaheuristic Algorithms*: Advanced heuristics for solving large, complex problems, including genetic algorithms and simulated annealing.

IV. **Hybrid Models**
Hybrid models combine multiple modeling approaches to address complex systems. For example, a hybrid model may use simulation to estimate demand variability and optimization to design a robust delivery network.

1.3 Key Objectives of Modeling

The goals of modeling in transport and distribution logistics depend on the problem domain. Key objectives include:

I. **Cost Minimization**
Models aim to reduce operational costs by optimizing transportation [17], storage, and inventory expenses. For instance:

- *Transportation Costs*: Optimizing delivery routes and selecting cost-effective transportation modes. [4], [5]
- *Inventory Costs*: Balancing holding costs against ordering costs through inventory-transportation models.

II. **Service Level Optimization**
High service levels are crucial for customer satisfaction. Models help ensure:

- *On-Time Delivery*: Minimizing delays by accounting for uncertainties like traffic or weather.
- *Product Availability:* Ensuring sufficient stock levels through coordinated inventory and distribution strategies.

III. **Environmental Impact Reduction**
Green logistics [8] models incorporate environmental factors, such as fuel efficiency and emissions. This involves:

- *Route Optimization*: Minimizing fuel consumption by choosing shorter or less congested routes.
- *Mode Selection*: Encouraging the use of sustainable modes like rail or electric vehicles.

IV. **Risk Mitigation and Resilience**
Advanced models address uncertainties and disruptions, such as demand surges or supply chain disruptions, by incorporating scenario planning and sensitivity analysis. [6]

1.4 Underlying Techniques

Several mathematical and computational techniques underpin these models:

- *Graph Theory*: Represents transportation networks [18] as graphs with nodes (e.g., locations) and edges (e.g., routes). Common in shortest-path algorithms.
- *Stochastic Programming*: Deals with uncertainty by incorporating probabilistic data, such as demand forecasts or fuel prices.
- *Multi-Objective Optimization*: Balances trade-offs between conflicting objectives, such as cost and service level, using Pareto efficiency.

2. APPLICATIONS AND EXAMPLES

2.1 Transport task

Formulation of a classic transport task

We have p suppliers $D_1, D_2, \dots, D_p$, with capacities $a_1, a_2, \dots, a_p$, and q of customers $O_1, O_2, \dots, O_q$, with requirements $b_1, b_2, \dots, b_q$. The cost of transporting a unit of goods from the supplier D_i to the customer O_j is d_{ij}. Our task is to determine how many goods to bring from which supplier to which customer in such a way that we deliver as many goods as possible so that neither the capacities of suppliers nor the requirements of customers are exceeded and so that the total price for transporting all goods is as low as possible

Definition: The task of linear programming is to find such real numbers $x_1, x_2, \dots, x_n$, for which $f(x)$ is minimal,
while
$f(x) = \mathbf{c}^T.\mathbf{x} = c_1x_1 + c_2x_2 + \cdots + c_nx_n,$
where
$\mathbf{c}^T = c_1, c_2, \dots, c_n$ a $\mathbf{x} = x_1, x_2, \dots, x_n,$
and we are ooking the minimum value of $f(x)$ under the assumptions:
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$
$$x_1, x_2, \dots, x_n \geq 0.$$

Briefly, using matrix notation, the linear programming task can be formulated as: Minimize $\mathbf{c}^T.\mathbf{x}$ under the assumptions
$\mathbf{A}.\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0.$
For solving the problem of lin
ear programming, we have, for example, the famous simplex method, which (or its modifications) can be used with today's computing technology to handle even problems with thousands of variables.

Other practical requirements may require that the variables $x_1, x_2, \dots, x_n$, represented the numbers of real objects. In this case, all variables must be $x_1, x_2, \dots, x_n$, integers. A special case is the case when we require that all variables take only the values 0 or 1. There are several mathematical models for the transport task. The linear programming model is as follows. Let's first assume that the sum of the customers' demands is equal to the sum of the suppliers' capacities, i.e.
$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j.$

Such a transport task is called *balanced*. Let X be a matrix of real numbers of the type p × q, whose element x_{ij} means the amount of goods imported from the supplier D_i to the customer O_j. Then the total price for the transportation of all goods will be $\sum_{i=1}^{p} \sum_{j=1}^{q} d_{ij}x_{ij}$. Solving the transportation task means determining the elements of the matrix X so that
$$\sum_{i=1}^{p} \sum_{j=1}^{q} d_{ij}x_{ij} \qquad (1)$$
was minimal under the assumptions

$\sum_{j=1}^{q} x_{ij} = a_i, \quad \forall\, i = 1,2,\cdots,p$ (2)
$\sum_{i=1}^{p} x_{ij} = b_j, \quad \forall\, j = 1,2,\cdots,q$ (3)
$x_{ij} \geq 0, \ \ \forall\, i = 1,2,\cdots,p\,, \quad \forall\, j = 1,2,\cdots,q$ (4)

The double sum in (1) means total transportation costs; condition (2) means that we will transport the entire offered quantity a_i from each supplier D_i; and condition (3) means that we will deliver all the required quantity of goods b_j to each customer. Condition (4) says that negative quantities of goods cannot be transported.
If the suppliers' offer was greater than the customers' requirements, i.e.

$\sum_{i=1}^{p} a_i > \sum_{j=1}^{q} b_j,$

the task model will be changed so that in conditions (2) instead of "=" the relation "≤", i.e., we will export at most a_i units of goods from each supplier. Other constraints (3) and (4) remain unchanged. Likewise, if

$\sum_{i=1}^{p} a_i < \sum_{j=1}^{q} b_j,$

then in conditions (3) the "=" relation "≤" changes.
Some methods of solving the traffic task assume its balanced shape. We will convert an unbalanced task into a balanced case so that the sum of the suppliers' capacities exceeds the sum of the customers' requirements, i.e. if

$\sum_{i=1}^{p} a_i > \sum_{j=1}^{q} b_j,$
then we supply the dummy customer O_{q+1} with the request

$b_{q+1} = \sum_{i=1}^{p} a_i {=} \sum_{j=1}^{q} b_j$

and all transport costs $d_{i(q+1)}$zero.
Analogously in the case

$\sum_{i=1}^{p} a_i < \sum_{j=1}^{q} b_j,$

we supply a fictitious supplier.

2.2 Green logistic modelling

Green logistics modeling is a structured approach to optimize logistics activities (such as transportation, warehousing, and inventory management) with an explicit focus on minimizing environmental impact while maintaining economic and service-level efficiency.
The mathematical definition of green logistics modeling typically involves multi-objective optimization, where both cost and environmental impact are considered as competing objectives.

General Mathematical Formulation
min ($f_1(x)$, $f_2(x)$,…, $f_k(x)$)
Where:

- $f_1(x)$: Represents total economic cost (e.g., transportation cost, warehousing cost).
- $f_2(x)$: Represents total environmental impact (e.g., carbon emissions, energy consumption).
- x: Decision variables, including routing decisions, mode selection, shipment quantities, and warehouse operations.
- k: Number of objectives (e.g., cost, emissions).

The solution satisfies:

- Constraints ($g_i(x) \leq 0,\ \forall i$): Representing practical limits like vehicle capacities, time windows, and regulatory standards.
- Non-negativity constraints ($x \geq 0$): Ensuring all variables are feasible in the physical system.

Specific Components in Green Logistics Modeling

Objective Functions

- **Economic Cost** ($f_1(x)$):

$f_1(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} C_{ij} x_{ij}$
where:

- C_{ij}: Cost of transporting goods from location i to j.
- x_{ij}: Quantity of goods transported between i and j.
- N ,M: Number of origin and destination nodes.

- **Environmental Impact** ($f_2(x)$):

$f_2(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} E_{ij} x_{ij}$
where:

- E_{ij}: Emissions or energy consumption per unit transported fromi to j.
- x_{ij}: Decision variables as defined above.

Constraints

- **Flow Balance**: Ensuring supply meets demand at all nodes:

$$\sum_{j=1}^{M} x_{ij} - \sum_{j=1}^{M} x_{ji} = S_i \ ,\forall i$$

where S_i is the net supply at node i (positive for supply, negative for demand).

- **Capacity Constraints**:

$x_{ij} \leq U_{ij}, \forall\ i,j$
where U_{ij} is the capacity of the transport mode or warehouse.

- **Emission Regulations**:

$\sum_{i=1}^{N} \sum_{j=1}^{M} E_{ij} x_{ij} \leq E_{max}$
where E_{max} is the allowable emissions threshold.

Multi-Objective Optimization

This involves solving trade-offs between cost and emissions, which can be achieved using methods such as:

- Weighted Sum Method:

min α·$f_1(x)$+β·$f_2(x)$,
where α,β are weights reflecting the relative importance of cost and environmental impact.

- Pareto Optimization: Identifying a set of solutions where no objective can be improved without worsening another.

2.3 The Vehicle Routing Problem

The Vehicle Routing Proble (VRP) is a combinatorial optimization problem that seeks to determine the optimal routes for a fleet of vehicles to deliver goods or services to a set of customers while satisfying certain constraints and minimizing associated costs. [1], [11], [13]

Key Components

1. **Graph Representation**:
   - The problem is typically represented on a graph $G=(V,E)$
     - $V=\{0,1,2,\ldots,n\}$: Set of vertices, where $v_0$represents the depot and v_1, v_2v2,…, v_n represent the customers.
     - E: Set of edges connecting the vertices, representing possible routes.
   - Each edge $(i,j)\in E(i, j)$ has an associated cost c_{ij}, often representing distance or travel time.
2. **Demand**:
   - Each customer i has a demand $d_i$with $d_i \geq 0$. The depot has $d_0 = 0$.
3. **Vehicles**:
   - A fleet of K vehicles, each with a maximum capacity Q, starts and ends at the depot v_0.
4. **Routes**:
   - A route is a sequence of vertices $R_k=\{v_0, v_{i_1}, v_{i_2},\ldots, v_{i_m}, v_0\}$ visited by vehicle k, where

$\sum_{i\in R_k} d_i \leq Q$.

Objective Function

The objective of VRP is to minimize the total cost of the routes, typically defined as:
Minimize $Z=\sum_{k=1}^{K} \sum_{(i,j)\in R_k} c_{ij} x_{ij}^k$
where:

- x_{ij}^k =1 if vehicle k travels directly from i to j, and 0 otherwise.
- c_{ij}: Cost (distance or time) of traveling from i to j.

Constraints

The VRP involves several constraints to ensure the feasibility of the solution:

1. **Each customer is visited exactly once:**

$$\sum_{k=1}^{K} \sum_{j\in V, j\neq i} x_{ij}^k = 1 \ \forall\, i \in V, i \neq 0$$

2. **Flow conservation at each vertex**: For each customer iii, the number of vehicles arriving must equal the number of vehicles leaving: $\sum_{j\in V, j\neq i} x_{ij}^k = \sum_{j\in V, j\neq i} x_{ji}^k, \ \forall\, k, i \in V$

3. **Capacity constraint for each vehicle**: The total demand served by a vehicle cannot exceed its capacity Q: $\sum_{i\in R_k} d_i \leq Q, \ \forall\, k$.

4. **Depot visit constraint**: Each vehicle starts and ends its route at the depot: $\sum_{j\in V, j\neq 0} x_{0j}^k = 1, \forall\, k$

$$\sum_{i\in V, i\neq 0} x_{i0}^k = 1, \forall\, k$$

5. **Subtour elimination constraint**: To prevent disconnected subroutes (subtours), additional constraints are required, such as the Miller-Tucker-Zemlin (MTZ) formulation:

$$u_i - u_j + Q\cdot x_{ij}^k \leq Q - d_j, \ \forall\, i,j \in V,\ i\neq j,\ k$$

where u_i is the cumulative demand at node i.

Sustainable Logistics Modeling
The need for environmentally sustainable logistics systems is a driving force behind many modeling innovations. Governments, businesses, and consumers are increasingly focused on reducing the environmental impact of transportation and distribution activities.

3. FUTURE TRENDS

The future of transport and distribution logistics modeling is shaped by rapid advancements in technology, increasing complexity in supply chains, and the growing need for sustainable practices. Here are the key areas driving innovation and development in the field:

Integration of Artificial Intelligence (AI) and Machine Learning
AI and machine learning (ML) technologies are revolutionizing logistics modeling by enabling systems to process and analyze vast amounts of data. These technologies offer predictive insights, real-time adaptability, and autonomous decision-making capabilities.

Case Study:
Amazon’s AI-driven logistics network leverages predictive analytics to optimize delivery routes and manage warehouse operations. A study by [16] estimated that such AI-driven strategies reduced last-mile delivery costs by 10–15%.

Real-Time Decision-Making with IoT

The Internet of Things (IoT) is another transformative trend in logistics modeling. IoT-enabled devices, such as GPS trackers, temperature sensors, and RFID tags, provide real-time visibility into supply chain operations. These devices generate continuous streams of data that can be integrated into predictive models to enhance decision-making.

Example:
A 2022 study by [10] demonstrated that IoT-based logistics networks achieved a 25% reduction in delivery delays by optimizing routes based on real-time data.

Sustainable Logistics Modeling
The need for environmentally sustainable logistics systems is a driving force behind many modeling innovations. Governments, businesses, and consumers are increasingly focused on reducing the environmental impact of transportation and distribution activities.

Case Study:
A 2023 study by [12] focused on a retail chain’s use of a multi-objective model to reduce greenhouse gas emissions. The model integrated renewable energy-powered warehouses and electric delivery vehicles, achieving a 20% reduction in emissions without compromising service quality.

Advances in Computational Techniques
Modern computational techniques are enhancing the ability to solve complex logistics problems more efficiently.

Example:
A logistics firm utilized cloud-based computing to simulate a nationwide distribution network. According to [14], this approach cut computation time by 50%, allowing for more frequent model updates and real-time scenario testing.

4. CONCLUSION

A key component of contemporary supply chain management is modeling in transportation and distribution logistics, which helps companies to reduce expenses, improve service quality, and solve environmental issues. Although theoretical models offer insightful information, their practical applications frequently encounter difficulties that call for creative solutions. Future advancements in AI, IoT, and sustainability-focused models have the potential to completely transform the discipline and present both researchers and practitioners with new opportunities.

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